FORMAT OF THE QUESTION PAPER

The Additional Mathematics 2 (3472/2) consists of 15 questions which are divided into three sections as follows:

Section A:	This section consists of 6 questions of medium length from the compulsory package of the syllabus. Candidates are required to answer all questions in this section.
Section B:	This section consists of 5 long questions from the compulsory package of the syllabus. Candidates are required to choose 4 questions from this section.
Section C:	This section consists of 4 long questions from the application package of the syllabus. Candidates are required to answer any 2 questions from this section.

The time allocated to answer this paper is 2 hours and 30 minutes.

This year's paper is still bilingual but the format is different from the past three years. Questions in English appear first followed by the equivalent bahasa Malaysia version in accordance to the format of "One After Another" which is different from the format of "Facing Pages" used before.

GENERAL PERFORMANCE

The performance of the candidates was expected to improve from the previous years. Nevertheless, as in previous years, the difference in performance was significant from centres to centres. Some centres performed extremely well while others did not perform that well. Candidates in the high achievement groups in some centres showed great improvements in their marks compared to the candidates in the same category last year.

Generally, candidates performed well in topics such as Simultaneous Equations, Coordinate Geometry, Arithmetic Progressions, Linear Law and Index Number compared to Calculus, Statistics, Vector and Motion Along A Straight Line.

PERFORMANCE ACCORDING TO CANDIDATES GROUP

Candidates in the High Achievement Group

The performance of the candidates in this group was expected to be better than before. In some centres, there was a significant improvement in the marks compared to the same group last year. Candidates were able to answer the questions accurately. Most candidates were able to answer more than 4 questions from Section B or more than 2 questions from Section C. Candidates' answers were complete, neat and systematic. They hardly made any mistake in solving the problems. They also understood and acquired the concepts in the topics that they answered.

Candidates in the Moderate Achievement Group

The performance of the candidates was satisfactory. Candidates were able to answer questions which had tested their basic skills, but they faced problems in the application type of questions. Candidates were unable to answer well in several questions such as 4, 5, 6(c) from Section A and questions 8(c), 10, 11(b) (ii), 13(c), and 15(b)(i). Some of the candidates were careless in their calculations or in simple algebraic manipulation.

Candidates in the Low Achievement Group

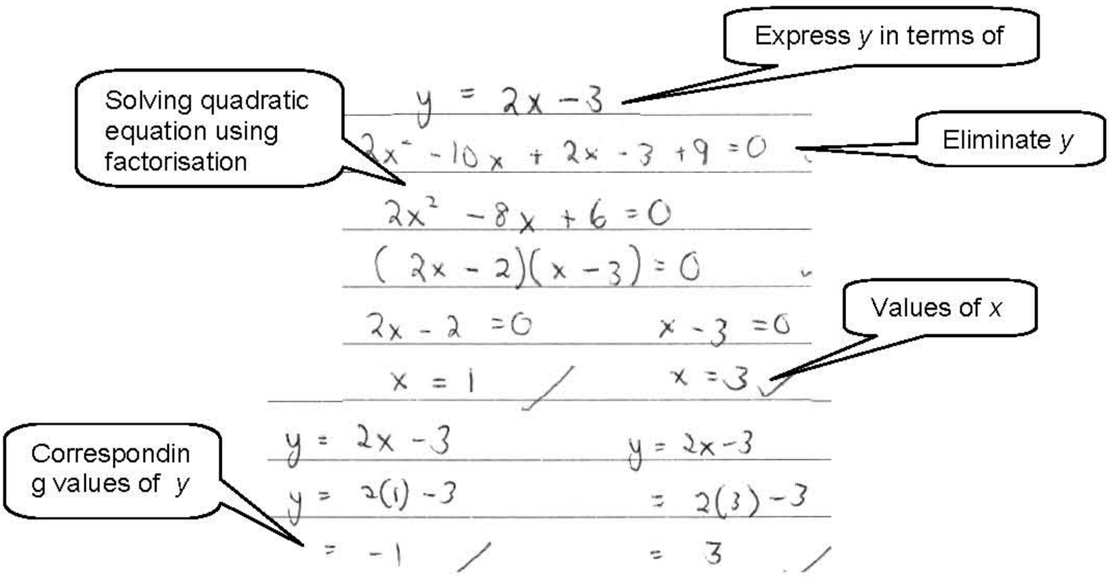
The performance of candidates was very weak. Most of the questions in Section A were not completely answered or not tried at all. Sometimes, the symbols used and workings were not related to the questions. Presentations of the answers were also not systematic. They were very weak in basic skills. Candidates made mistakes when solving questions involving operations of fractions, expansion and algebraic expression. The application of formulae was very weak. Although the list of formulae was given in the question paper, the candidates still wrongly wrote the formulae. Candidates showed a minimal understanding of the symbols in the formulae.

DETAILED PERFORMANCE

Question 1

The majority of the candidates managed to answer this question well.

This question tested the skills of solving simultaneous equations. Candidates mastered the methods of solving simultaneous equations involving linear and non-linear equations. Candidates could express one unknown in terms of the other unknown, and then eliminated one unknown. Candidates could solve quadratic equations using factorisation or quadratic formula. They were then able to find the values of the second unknown.



Some of the candidates used the elimination method.

$2x - y - 3 = 0$

$2x^2 - 10x + y + 9 = 0$

① + ②

$2x^2 - 8x + 6 = 0$

Add the two equations

There were a few candidates who were still weak in the algebraic manipulation, particularly, in handle '+ / –' sign.

$2x = y - 3$

$x = \frac{y - 3}{2}$

$2x - y - 3 = 0$
Then, it should be

Some candidates were weak in algebraic expressions, especially in algebraic expansion.

It should be
$2\left(\frac{y-3}{2}\right)^2 = 2\frac{(y-3)^2}{2^2}$

$\cancel{2}\frac{(y-3)^2}{\cancel{2}} - \cancel{10}^{5}\frac{(y-3)}{\cancel{2}_1} + y + 9 = 0$

Quite a number of candidates did not show the methods of solving quadratic equation. They gave the correct answers without showing the factorisation or using the quadratic formula or completing the square.

$2x^2 - 10x + 2x - 3 + 9$

$2x^2 - 8x + 6 = 0$

$x = 3$ @ $x = 1$

Methods to solve quadratic equation such as factorisation, using formula or completing the square must be shown.

There were still candidates who did not find the values of the second unknown.

(i)

$(y + 1)(y - 3) = 0$

$y = -1$ or 3

Values of *x* should be calculated

(ii)

$x^2 - 4x + 3 = 0$

$(x - 3)(x - 1)$

$x = 3, \; x = 1$

Values of *y* should be calculated

There were also candidates who were careless in their calculations.

(i)

$$2x^2 - 10x + 2x - 3 + 9 = 0$$
$$2x^2 - 8x - 6 = 0$$

– 3 + 9 = +

(ii)

$$y = 2(1) - 3$$
$$= 0$$

2(1) – 3 = – 1

Some candidates were careless while writing their solutions.

(i)

$$2x^2 - 10x + y + 9 = 0 \quad -②$$
$$① \text{ in } ②$$
$$2x^2 - 10 + (2x - 3) + 9 = 0$$

It should be `– 10*x*’

(ii)

Left out power of 2

$$2\left(\frac{y+3}{2}\right) - 10\left(\frac{y+3}{2}\right) + y + 9 = 0$$

Question 2

In question 2(a), candidates managed to find the *x*-intercept and *y*-intercept or managed to find the coordinates of *A* and *B.* Then, they managed to use the formula $(\frac{nx_1 + mx_2}{m+n}, \frac{ny_1 + my_2}{m+n})$ to find the coordinates of *P.*

In question 2(b), candidates were able to find the gradient of *AB*, m_1, from the equation $y + 2x + 8 = 0$ or from the points *A* and *B*. Candidates could also use the concept of perpendicular gradient, that is, $m_1 \times m_2 = -1$, where m_2 is the perpendicular gradient. Then they were able to find the equation of the perpendicular line by using the formulae $y - y_1 = m_2(x - x_1)$ or $y = m_2 x + c$.

(a) the coordinates of P :

when $x = 0$,

Know that the y-intercept is when $x = 0$ and state the

$y + 2(0) + 8 = 0$

$y = -8$ $\quad \therefore B(0, -8)$

when $y = 0$,

Know that the x-intercept is when $y = 0$ and state the

$0 + 2x + 8 = 0$

$2x = -8$ $\quad \therefore A(-4, 0)$

$x = -4$

$$(x, y) = \left(\frac{nx_1 + mx_2}{m+n}, \frac{ny_1 + my_2}{m+n} \right)$$

$$= \left(\frac{3(-4) + 1(0)}{4}, \frac{3(0) + 1(-8)}{4} \right)$$

Substitute the corresponding values into the formula to find the coordinates of *P*

$$= (-3, -2) \quad \therefore P(-3, -2)$$

Convert the equation into $y = mx + c$ to get the gradient of *AB*

b) $m_{AB} = -2$ $\quad y + 2x + 8 = 0$

$m_1 m_2 = -1$ $\quad y = -2x - 8$

$-2 m_2 = -1$ $\quad y - y_1 = m(x - x_1)$

$m_2 = \frac{1}{2}$ M1 $\quad y - (-2) = \frac{1}{2}(x - (-3))$

Subtitute the coordinates of *P* and m_2 into the equation of the straight line

Use $m_1 m_2 = -1$

$y + \cancel{4}\,2 = \frac{1}{2}x + \frac{3}{2}$

$y = \frac{1}{2}x + \frac{3}{2} - \cancel{4}\,2$

N1

$y = \frac{1}{2}x - \frac{1}{2} \rightarrow$ equation of the straight line

Quite a number of candidates used the wrong ratio when applying the formula for segment of a line, instead of using the ratio 1:3, they used 3:1.

$$(x, y) = \left(\frac{nx + mx}{m+n}, \frac{ny + my}{m+n} \right)$$

$$= \left(\frac{1(-4) + 3(0)}{4}, \frac{1(0) + 3(-8)}{4} \right)$$

It should be $m = 1$ and $n =$

Some of the candidates failed to find the coordinates of *A* or *B*. Candidates used the midpoint formula to find the coordinates of *P*.

$$P = \left(\frac{-4+0}{2}, \frac{0-2}{2} \right)$$

Should use the formula
$$\left(\frac{nx_1 + mx_2}{m+n}, \frac{ny_1 + my_2}{m+n} \right)$$

In question 2(b), a few candidates made mistakes while finding m_2 although they made used of the perpendicular gradient $m_1 \times m_2 = -1$.

$$m_1 m_2 = -1$$
$$-2 m_2 = -1$$
$$m_2 = 2$$

Should be
$$m = \frac{1}{2}$$

There were candidates who did not use point *P* to find the equation of the perpendicular line but they used $A(-4,\ 0)$ or $B(0,\ -8)$ instead.

$$y - 0 = 1/2(x + 4)$$
$$y = 1/2\,x + 2$$

Should use
P(-3, -2)

Some candidates used the gradient of the straight line *AB* instead of the perpendicular gradient.

$$y + 2 = -2(x+3)$$
$$y = -2x - 8$$

Should use perpendicular gradient, $m_2 = \frac{1}{2}$

The coordinates of *x* and *y* were interchanged in the equation of the straight line.

∴ equation of perpendicular
$$= y + 3 = 1/2\,(x + 2)$$
$$y + 3 = 1/2\,x + 1$$
$$y = 1/2\,x - 2$$

Should be
$$y + 2 = \frac{1}{2}(x + 3)$$

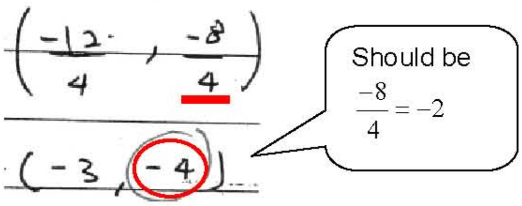
There were still some candidates who were careless in their calculations and also in writing the answers.

(i)

$$\left(\frac{-12}{4}, \frac{-8}{4} \right)$$
$$= (-3, 2)$$

Should be
$$\frac{-8}{4} = -2$$

(ii)



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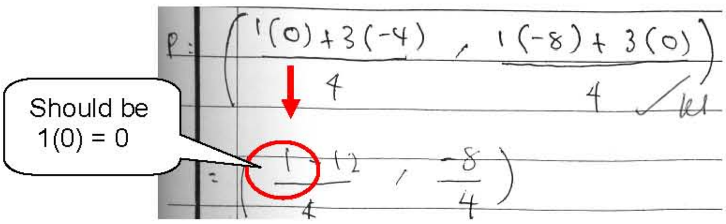

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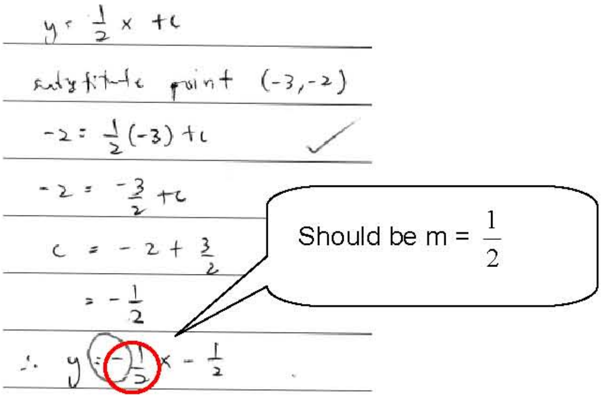
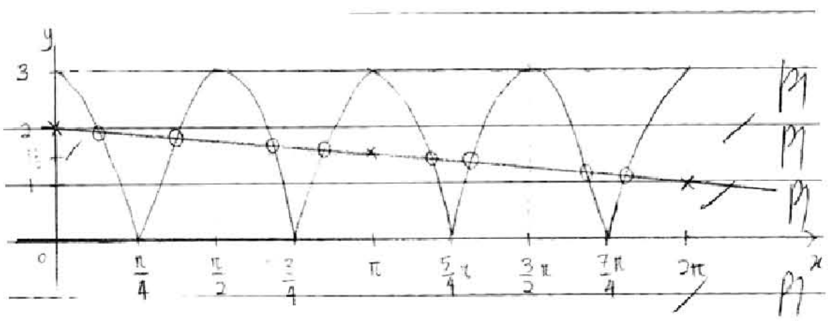


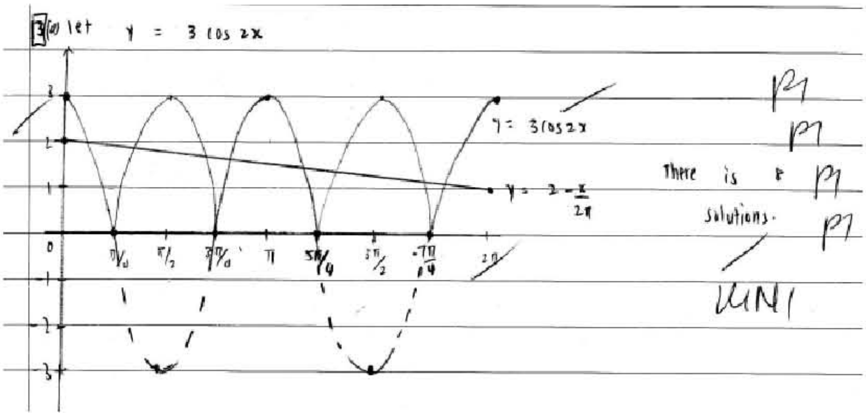
Question 3(a)

Candidates in the higher achivement group were able to sketch the graph of $y = |3\cos 2x|$ correctly. They were able to sketch the shape of the cosine graph. They knew that '2 in 2*x*' means two periods for $0 \le x \le 2\pi$ and '3', which is the amplitude of the graph. They also knew the symbol '| |' means modulus that is the reflection of the negative part of the graph on the *x*-axis.

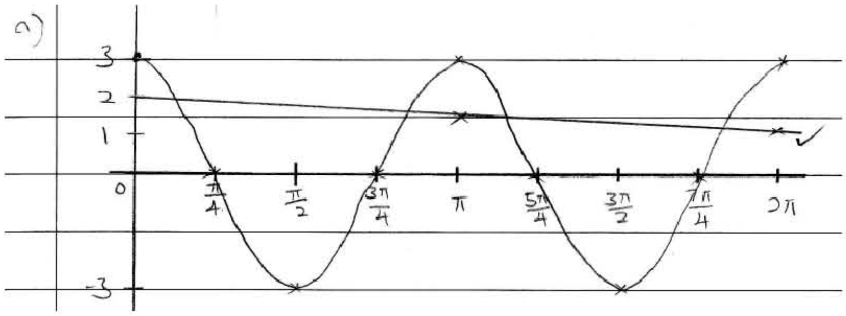
(i)



(ii)

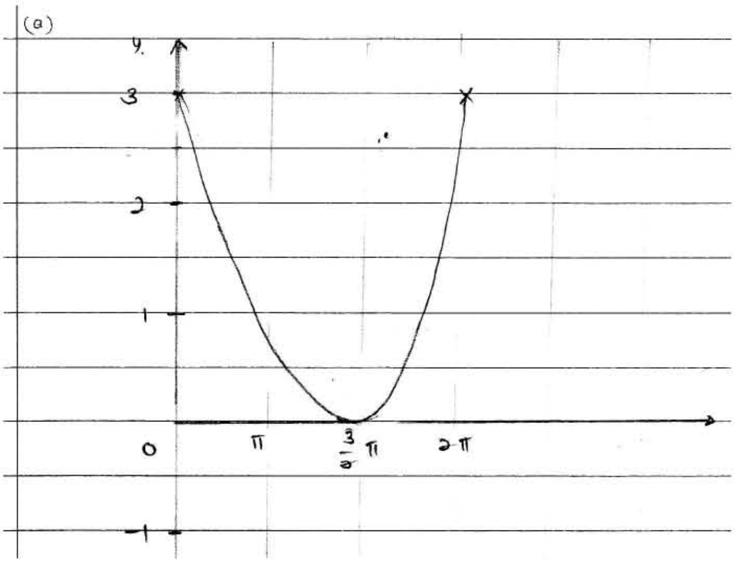


Quite a number of candidates managed to sketch the graph of $y = 3\,cos\,2x$, without the modulus part.



A few candidates could not even sketch the shape of cosine graph as seen in the examples below.

(i)



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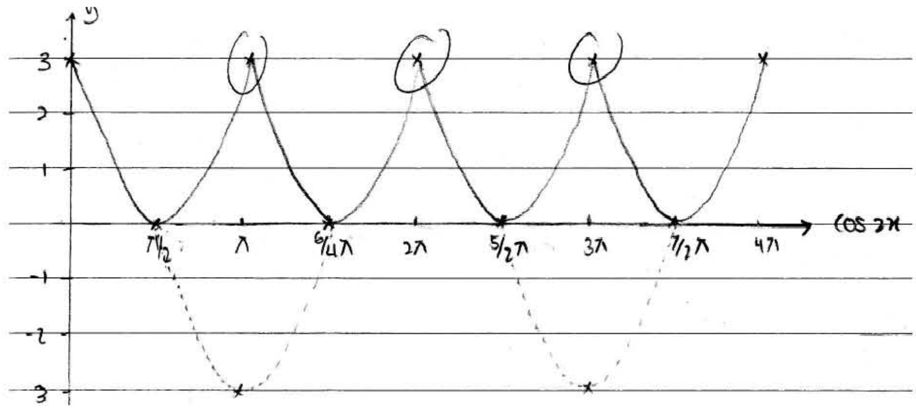
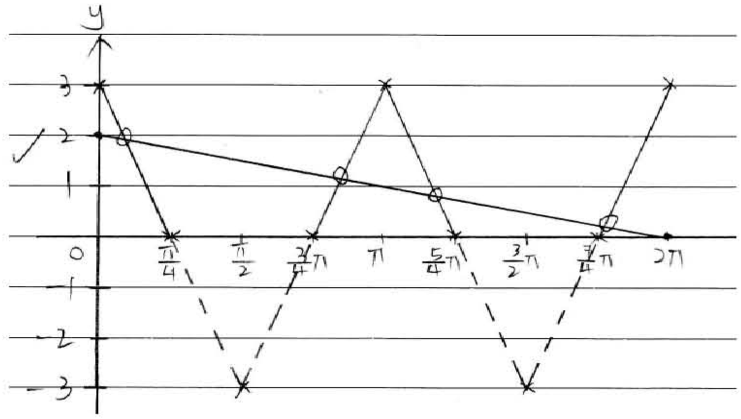
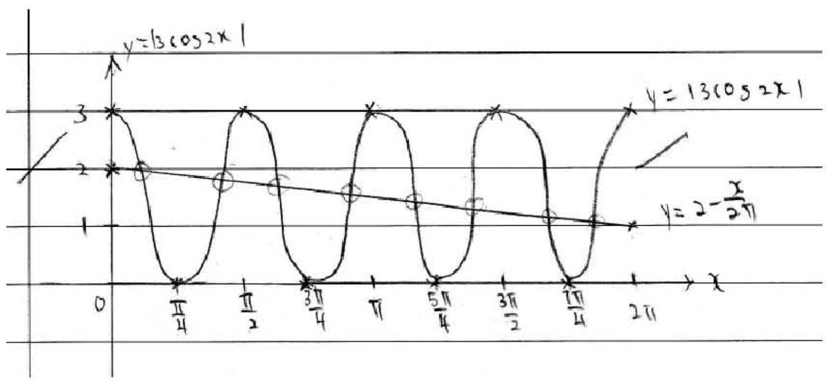


(iii)

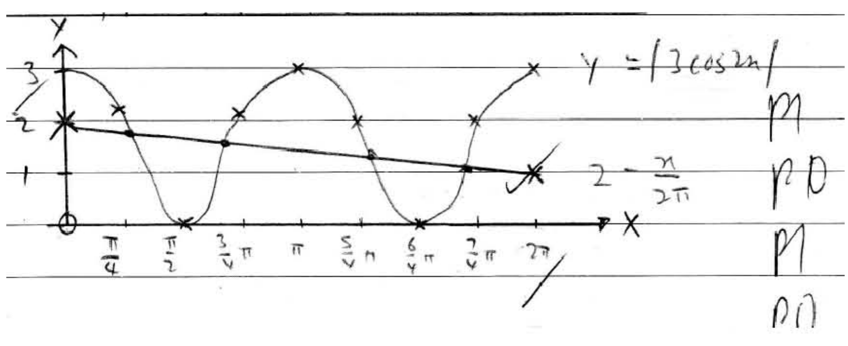


Some candidates could sketch the cosine graph but had wrongly interpreted the symbol of '| |'. They shifted the graph instead of reflecting the negative part of the graph on the *x*-axis.

(i)

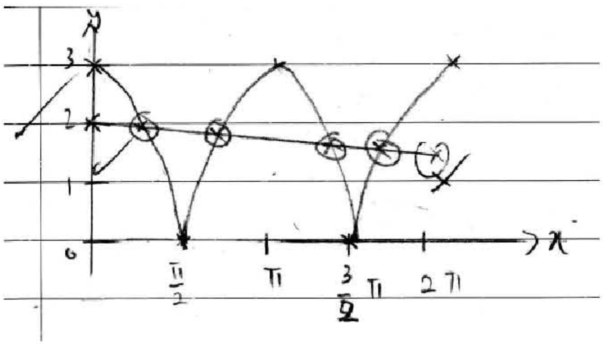


(ii)

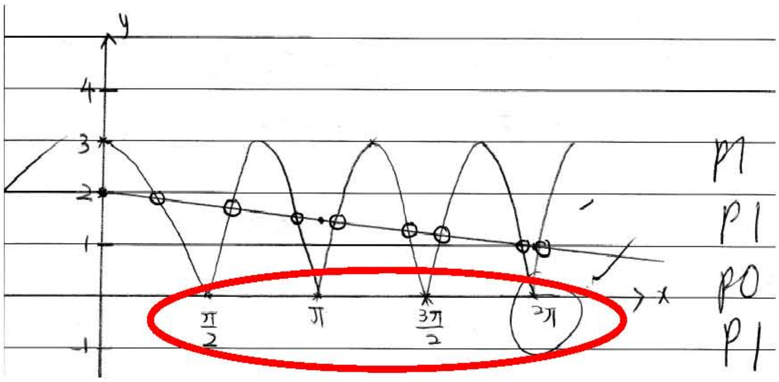


Some candidates could sketch the modulus cosine graph but did not know the meaning of '2 in 2*x*' as two periods for $0 \leq x \leq 2\pi$. They sketched a graph of $y = |3 \cos x|$ as shown in (i) or a graph of $y = \left|3 \cos 1\frac{3}{4}x\right|$ as shown in (ii).

(i)

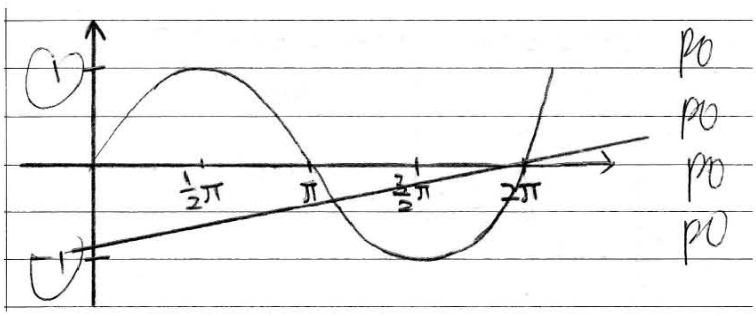


(ii)

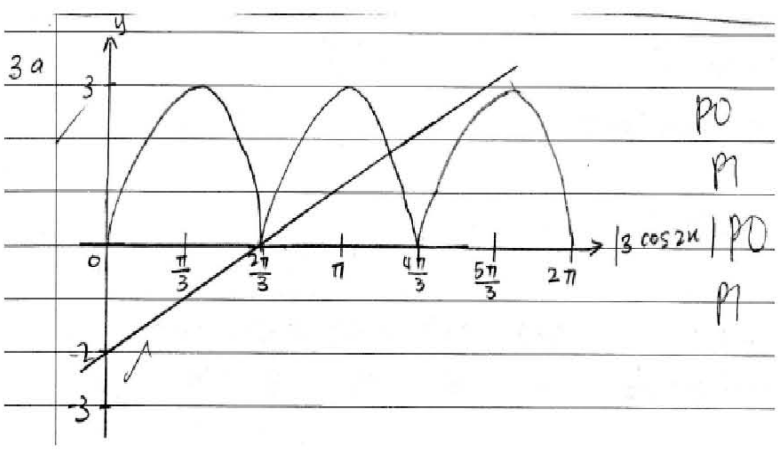


There were a few candidates who sketched a sine graph instead of the cosine graph.

(i)

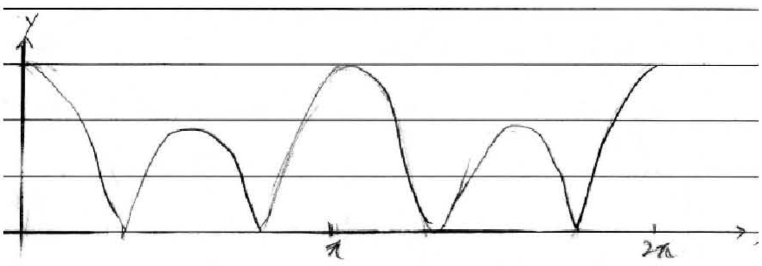


(ii)

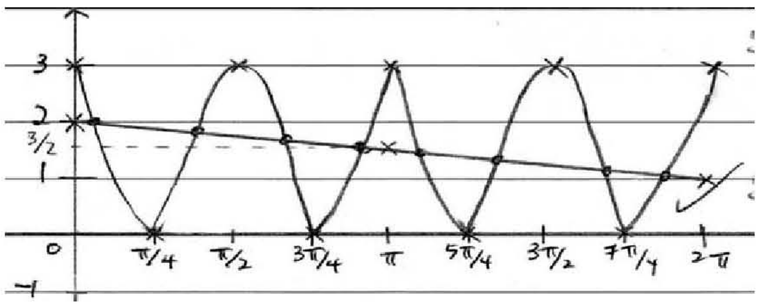


The shapes of the cosine graph that some of the candidates sketched were not perfect such as the amplitudes were not uniform as in example (i). Certain parts of the graph were concave instead of convex as shown in examples (ii) and (iii).

(i)



(ii)



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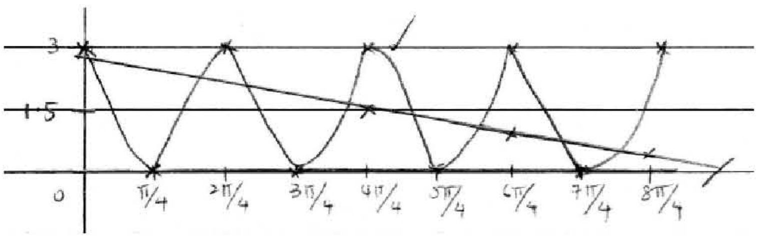
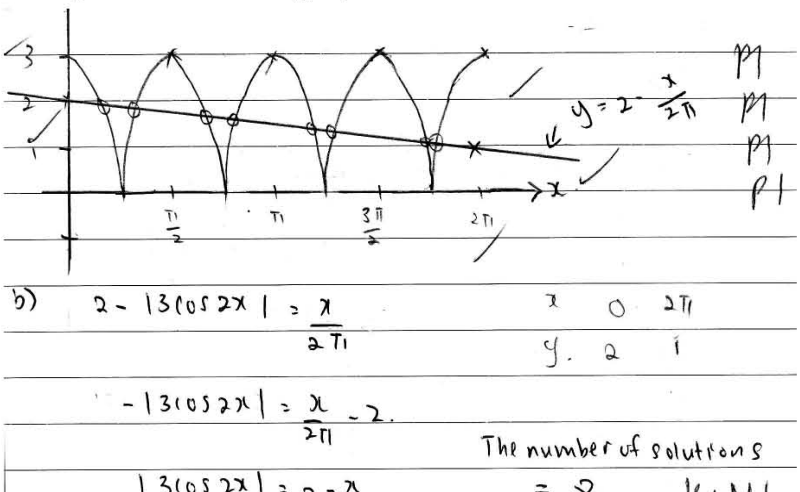


Question 3(b)

Candidates were able to find the equation of the straight line and sketched the straight line correctly. Candidates were also able to determine the number of solutions from the intersection points of the two graphs.



However candidates in the low achivement group could not relate the equation $2-|3\cos 2x| = \frac{x}{2x}$ to the equation of the graph in 3(a). Therefore they could not get the suitable equation of the straight line.

(i)

b) $2 - |3\cos 2x| = \frac{x}{2\pi}.$

$2 - 3 - 6\sin^2 x = \frac{x}{2\pi}$

$-1 - 6\sin^2 x = \frac{x}{2\pi}$ MO

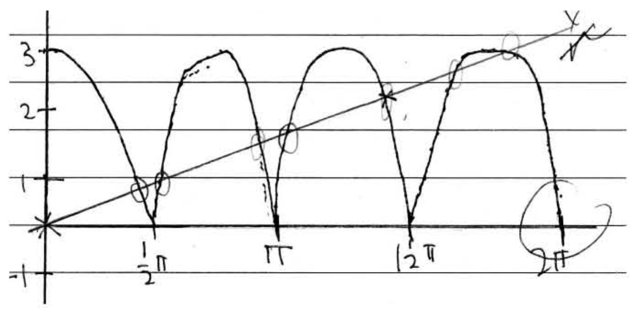
(ii)

$2 - |3\cos 2x| = \frac{x}{2\pi}$ MO

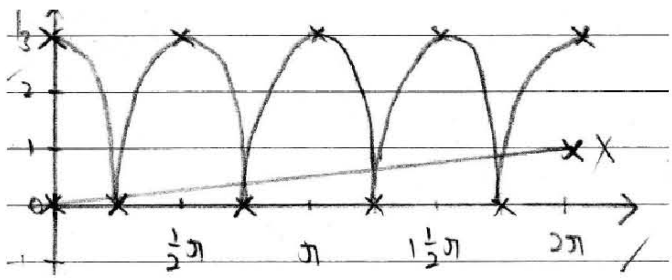
$2 - |3\cos 2(90)| = \frac{90}{2\pi}$

Eventhough the candidates managed to get the equation of the straight line, they had failed to draw the straight line, neither the *y*-intercept nor the gradient was correct.

(i)



(ii)



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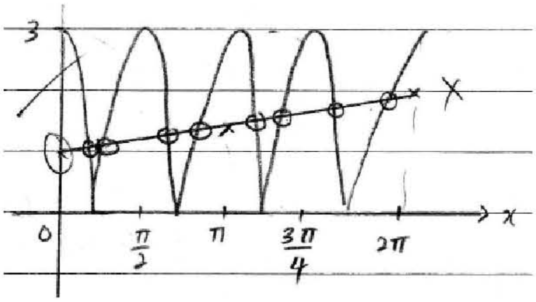

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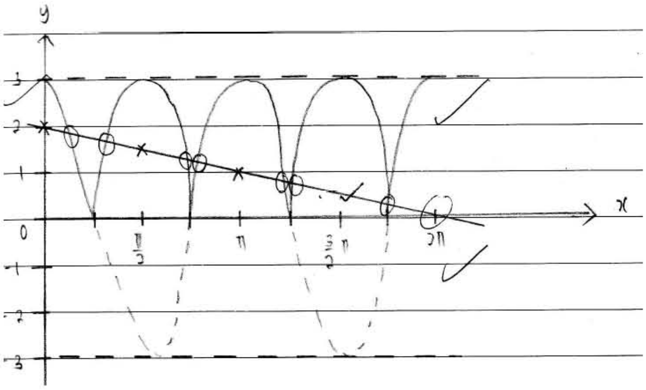


There were still a few candidates who were careless in the basic manipulation of algebra such as leaving out the '–' sign.

(i)

$2 - |3\cos 2x| = \frac{x}{2\pi}$

$|3\cos 2x| = \frac{x}{2\pi} - 2$

(ii)

b) $2 - y = \frac{x}{2\pi}$

$y = \frac{x}{2\pi}\ (-)2$ M0

Question 4(a)

Candidates in the high achievement group excellently answered this question.

Candidates knew the gradient function is $\frac{dy}{dx}$. Therefore, they used $\frac{dy}{dx} = 2x - \frac{2}{x^2}$ and equated to zero, showing that they knew the concept of turning point. Then they were able to solve correctly the value of k.

(i)

4) $\frac{dy}{dx} = 2x - \frac{2}{x^2}$ P1

a) At turning point $(k, 8)$, $\frac{dy}{dx} = 0$

$0 = 2k - \frac{2}{k^2}$ M1

$= 2k^3 - 2$

$= k^3 - 1$

$k^3 = 1$

$k = 1$ M1

(ii)

4) a) $\frac{dy}{dx} = 2x - \frac{2}{x^2}$ ✓ P1

$x = k$, $\frac{dy}{dx} = 2k - \frac{2}{k^2}$

$\frac{dy}{dx} = 0$, $2k - \frac{2}{k^2} = 0$

$2k = \frac{2}{k^2}$ ✓ K1

$k^3 = 1$

$k = 1$ ✓ N1

There were a few candidates who got the answer $x = 1$ but did not write $k = 1$ as the final answer.

4. (a) $2x - \frac{2}{x^2} = 0$ ✓ $(k, 8)$ P1

$2x^3 - 2 = x^2$ K1

$2x^3 - x^2 = 2$

$2x = 2$

$x = 1$ N0

There were also candidates who knew the gradient function is $\frac{dy}{dx}$ and the concept of turning point but they were not able to complete the solution.

$\frac{dy}{dx} = 2x - 2x^{-2}$ ✓ P1

At the turning point, $\frac{dy}{dx} = 0$.

$2x - 2x^{-2} = 0$

K0 N0

There were some candidates who did not know the meaning of gradient function and the concept of turning point. They assumed that $y = 2x - \frac{2}{x^2}$ and equated to 8 from the turning point (k, 8).

(i)

(4) (a) $y = 2x - \frac{2}{x^2}$

Given that $(k, 8)$

$8 = 2k - \frac{2}{k^2}$

(ii)

$y = 2x - \frac{2}{x^2}$

$8 = 2x - \frac{2}{x^2}$

Some candidates knew the concept of turning point, that is $\frac{dy}{dx} = 0$, but wrongly interpreted the gradient function as $y = 2x - \frac{2}{x^2}$. They could differentiate to get $\frac{dy}{dx}$ as shown in (i). However, there were candidates who used integration instead of differentiation of the function $y = 2x - \frac{2}{x^2}$ to get $\frac{dy}{dx}$ as shown in (ii).

(i)

$$\frac{dy}{dx} = 2 - 4x^{-3} \quad \text{P0}$$

$$2 - \frac{4}{x^3} = 0 \quad \text{K0}$$

(ii)

$$\frac{dy}{dx} = \frac{2x^2}{2} - \frac{2x}{x^3}$$

$$0 = x^2 - \frac{2}{x}$$

Meanwhile, there were quite a number of candidates in the low achievement group who did not know the concept of turning point and the meaning of gradient function.

(i)

$$\text{(a)} \quad \int 2x - \frac{2}{x^2}\, dx = \int 2x - 2x^{-2}\, dx \quad \text{P0}$$

$$= \cancel{\int} x^2 + x^{-1} \cancel{dx} + c. \quad \text{K0}$$

$$= x^2 + x + \left(\tfrac{1}{2}\right)^2 - \left(\tfrac{1}{2}\right)^2 + c$$

$$= \left(x + \tfrac{1}{2}\right)^2 - \tfrac{1}{4} + c$$

$$k = \tfrac{1}{2} \quad \text{N0}$$

(ii)

$$\frac{dy}{dx} = 2 - \frac{4}{x^3}$$

$$8 = 2 - \frac{4}{x^3}$$

(iii)

$$dy/dx = 1 + 2x \quad \text{P0}$$

$$\text{when } y = 8, \quad 8 = 1 + 2x \quad \text{K0}$$

$$8 - 1 = 2x$$

There were still many candidates who were careless with the algebraic manipulations as shown below.

(i)

$$\text{let } 2x - \frac{2}{x^2} = 0 \quad ✓$$

$$2x - 2 = x^2$$

(ii)

$$2x - \frac{2}{x^2} = 0 \quad ✓$$

$$2x - 2 = x^2$$

(iii)

$2x - \frac{2}{x^2} = 0$ ✓

$\frac{2x^3 - 2}{x^2} = 0$

$2x^3 - 2 = x^2$

(iv)

$\frac{dy}{dx} = 2x - \frac{2}{x^2}$

$2x - \frac{2}{x^2} = 0$

$2x - 2 = 0$

$2x = 2$

$x = 1$

$\therefore k = 1$

(v)

$0 = 2x - \frac{2}{x^2}$

$\frac{2}{x^2} = 2x$

$2 = 2x^3$

$x^3 = 1$

$x = \pm 1$ k..

(vi)

$x^2(2x) = 2$

$2x^3 = 2$

$x^3 = \frac{2}{2}$

$= 1$

$x = \frac{1}{3}$

$k = \frac{1}{3}$

Question 4(b)

Most candidates had determined the maximum or minimum point by finding $\frac{d^2y}{dx^2}$ and subtituted the value of x in $\frac{d^2y}{dx^2}$. They managed to get the value of $\frac{d^2y}{dx^2}$ to be 6 (>0) and therefore able to make the conclusion that the point was a minimum point.

(i)

b) $\frac{dy}{dx} = 2x - \frac{2}{x^2}$

$\frac{d^2y}{dx^2} = 2 + \frac{4}{x^3}$ ✓ $(x=1)$ K1

$= 2 + \frac{4}{1}$ ✓ K1

$= 6$ (min N1

the turning point is minimum point. ✓

(ii)

b) $\frac{dy}{dx} = 2x - 2x^{-2}$

$\frac{d^2y}{dx^2} = 2 + 4x^{-3}$ ✓ K1

$= 2 + \frac{4}{x^3}$

when $x = 1$, $\frac{d^2y}{dx^2} = 2 + \frac{4}{(1)^3}$ ✓ K1

$= 6$ (minimum)

$(1,8)$ is the minimum point. ✓ N1

However, some candidates did not use a valid method to determine the maximum or minimum point.

(i)

b) $\int 2x - \frac{2}{x^2} \quad dx.$

~~x^2~~ $- \int 2x - 2x^{-2} \quad dx.$ K0

$\left[x^2 - \frac{2x^{-1}}{-1} \right] \cancel{x}$

$= \left[x^2 + \frac{2}{x} + c \right]$

Since x^2 is positive, then the turning point is a minimum point.

N0

(ii)

b) $\frac{dy}{dx} = 2x - 2(x^{-2})$

$\frac{d^2y}{dx^2} = 2 + \frac{4}{x^3}$ K1

$-2 = \frac{4}{x^3}$

$x^3 = -2.$ N0

$x = -1.26 < 0$, it is a maximum point

There were a few candidates who had guessed the answer without showing the proper working method.

(i)

(b) ~~max~~ It is a maximum point.

(ii)

(b) the turning point is minimum point.

Some candidates gave the answers based on the wrong concepts.

(i)

(b) turning point $(\frac{2}{3}, 8)$ is a minimum point as the values are positive.

(ii)

(b) $y = x^2 + x + c$

$a = 1 > 0$ KO

$\therefore$ ~~maxim~~ turning point = maximum point. MO

(iii)

The turning point is a maximum point.

the gradient is $\frac{-2}{x^2}$ KO MO

(iv)

b) The turning point is a ~~minimum~~ maximum point because

it is smaller than 0.

A few candidates could not differentiate $2x - \frac{2}{x^2}$ correctly to find $\frac{d^2y}{dx^2}$ as shown in the examples below.

(i)

$\frac{dy}{dx} = 2x - \frac{2}{x^2}$

$\frac{d^2y}{dx^2} = 2 + \cancel{2}\, 4(x^{-1})$

(ii)

(b) $\frac{dy}{dn} = 2n - \frac{2}{n^2}$

$\frac{d^2y}{dn^2} = 2 (-) \frac{4}{x^3}$

(iii)

$\frac{dy^2}{d^2x} = 2 - 2x^{-2-1}$

$= 2 - 2x^{-3}$ K1

$= 2 (-) \frac{2}{x^3} \qquad = 2 (-) \frac{2}{(-1)^3}$

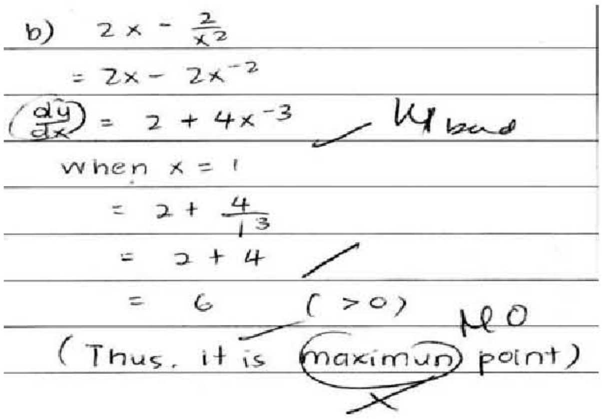
~~(1, 0) = 2 - 2~~ $= 2 + 2$

$= 4$ N0

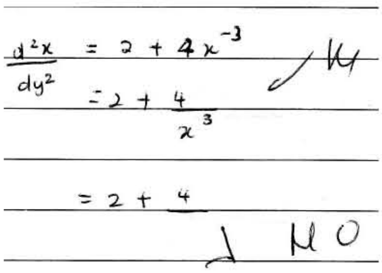
Turning point is a ~~maximum~~ minimum point.

Some candidates could not make either the correct conclusion as in (i) or any conclusion as shown in (ii), even though they got the value of $\frac{d^2y}{dx^2}$.

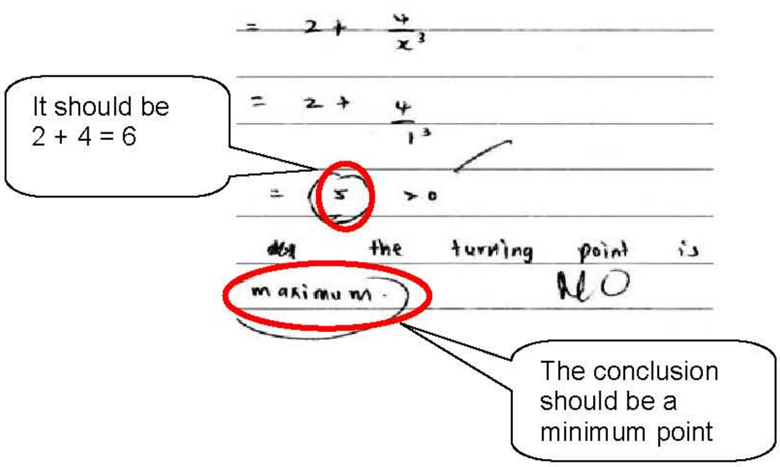
(i)



(ii)

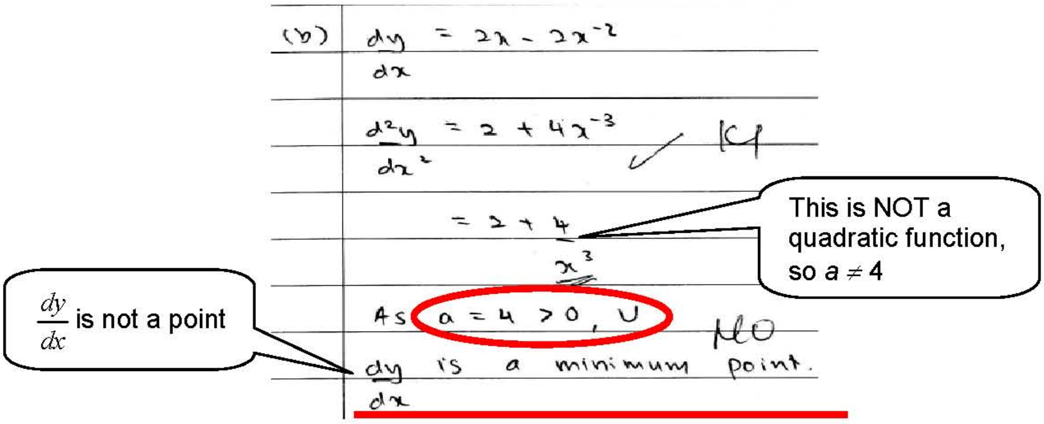


A careless mistake was seen in the calculation to find the value of $\frac{d^2y}{dx^2}$ eventhough the value of x was correctly substituted.



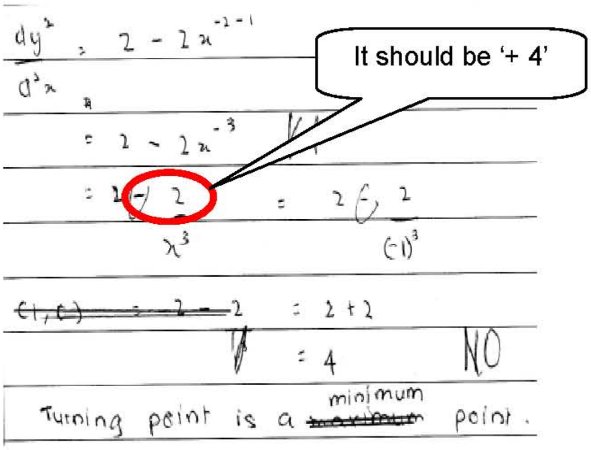
A few candidates did not substitute the value of x into $\frac{d^2y}{dx^2}$ and also made a conclusion based on a wrong concept.

(i)

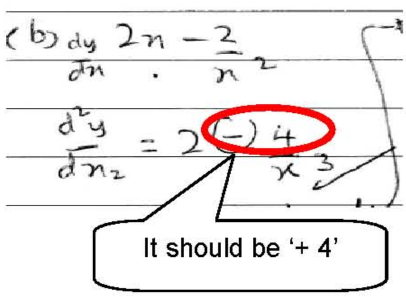


Some candidates made mistakes in the negative or positive sign and the coefficient value for $\frac{d^2y}{dx^2}$.

(i)

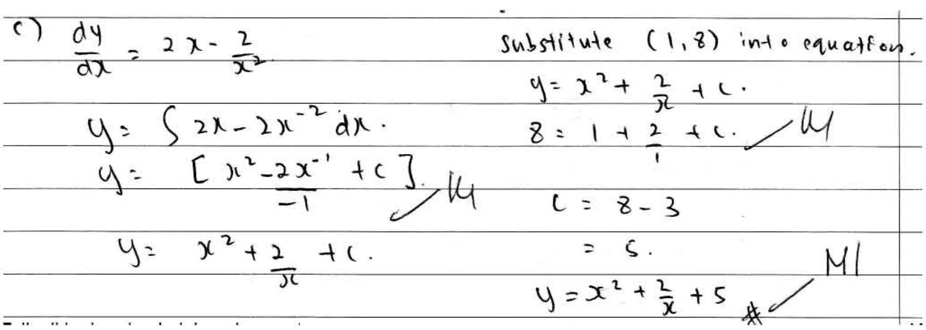


(ii)



Question 4(c)

There were quite a number of candidates who knew how to find the equation of the curve by integrating the gradient function and substituting the point (1, 8) to get the value of the constant, c. Hence, they were successful in getting the equation of the curve.



However, some candidates were not able to integrate $2x - \frac{2}{x^2}$ correctly.

(i)

$$\frac{dy}{dx} = 2x - 2x^{-2}$$

$$y = x^2 + 2x + c$$

(ii)

$$\int 2x - 2x^{-2}\ dx$$

$$= \int \frac{2x^2}{2} + \frac{2}{3}x^{-3}\ dx$$

$$y = x^2 + \frac{2}{3x^3}$$

(iii)

$$\text{(c)}\ y = \int 2x - \frac{2}{x^3}\ dx$$

$$y = x^2 + \frac{4}{x^3} + c$$

A few candidates used an invalid method to find the equation of the curve. Some used $\frac{dy}{dx} = 0$ as shown in (i) and (ii) while some used equation of the straight line as seen in (iii) to (vii).

(i)

Question 4c

$$2x - \frac{2}{x^2} = 0$$

$$2x - 2 = x^2$$

$$x^2 - 2x + 2 = 0$$

K0 K0

N0

(ii)

$$4)\ c)\quad 2x - \frac{2}{x^2} = 0$$

$$2x - 2 = -x^2$$

$$x^2 + 2x - 2 = 0$$

(iii)

$$\text{(c)}\quad \frac{y-8}{x-4} = -\frac{1}{4}$$

$$4y - 32 = -x + 4$$

$$x + 4y - 36 = 0$$

(iv)

$$\text{c.}\quad y = mx + c \qquad (-4, 8)$$

$$m = {}^{-}y/x$$

$$= {}^{-}8/{-4}$$

$$= 2$$

$$8 = 2(-4) + c$$

$$8 = -8 + c$$

$$c = -8 - 8$$

$$= -16$$

K0 K0

$\therefore$ The equation is $y = 2x - 16$ N0

(v)

$y_1 - y_2 = m(x_1 - x_2)$

$(y_1 - 8) = 2(x - (-2))$

(vi)

$y - y_1 = m(x - x_1)$

$y - 8 = 1/2(x - 2/7)$

$y = 1/2\,x - 1/7 + 8$

$y = 1/2\,x + 7\,6/7$

(vii)

For point $(1, 8)$, $\frac{dy}{dx} = 2(1) - \frac{2}{1^2}$

$= 0$

$\therefore$ Equation $= y - 8 = 0(x - 1)$ K0 K0

$y - 8 = 0$

$y = 8$ #

N0

Some candidates did not write the constant, *c*, in the integral of $2x - \dfrac{2}{x^2}$ as shown in (i) and (ii) or did not find *c* as shown in (iii)

(i)

Equation of curve

$\int 2x - \frac{2}{x^2}$

$= x^2 - \frac{2}{x}$ K1

K0 N0

(ii)

c) $\int (2x - 2x^{-2})\,dx$

$y = x^2 + \frac{2}{x}$ K1

#

K0 N0

(iii)

(c) $y = \int 2x - \frac{2}{x^2}\,dx$

$= \left[\frac{2x^2}{2} - \frac{2x^{-1}}{-1} + c\right]$ K1

$= x^2 + \frac{2}{x} + c$ K0

N0

There were still candidates who made mistakes in some parts of the integration.

(i)

$y = \int 2x - 2x^{-2}\,dx$

$= 4x^{-1} + \frac{\cancel{2}x^2}{\cancel{2}} + c$ K1

$y = \frac{4}{x} + x^2 + c$ $\qquad$ $y = \frac{4}{x} + x^2 + 3$

$8 = \frac{4}{1} + (1)^2 + c$ K1

$c = 3$

(ii)

c) Equation of the curve is

$y = \int 2x - \frac{2}{x^2}\,dx$

$= x^2 + \frac{4}{x} + c$ K1

At the point (1, 8)

$8 = (1)^2 + \frac{4}{(1)} + c$ K1

$c = 3$

$\therefore y = x^2 + \frac{4}{x} + 3$ # N0

Some candidates did not find the value of the constant of integration *c*, but simply used *c* = 8.

(i)

$y = \int (2x - 2x^{-2})\,dx$

$= \frac{2x^2}{2} - \frac{2x^{-1}}{-1} + c$ K1

$= x^2 + \frac{2}{x} + c$

K0

$= x^2 + \frac{2}{x} + 8$ N0

(ii)

c) eqn of curve

$= \int 2x - 2x^{-2}\,dx$

$= x^2 - \frac{2x^{-1}}{-1} + c$ K1

K0

$y = x^2 + \frac{2}{x} + 8$ N0

Question 5(a)

Some of the candidates could complete Table 2 correctly based on the given cumulative frequency distribution because they realised that the number of students in Table 1 is the cumulative frequencies.

a)	Score	0 – 9	10 – 19	20 – 29	30 – 39	40 – 49
	No. of students	4	6	10	8	4

Many candidates did not realise that the number of students in Table 1 is the cumulative frequency and took directly the values as the frequency.

Score	0 – 9	10 – 19	20 – 29	30 – 39	40 – 49
Number of students	4	10	20	28	32

Question 5(b)
Only a few candidates could adapt correctly the formula for median to find the first and third quartiles.

b) $Q_3 - Q_1$

$$\left[29.5 + \left(\frac{24-20}{8}\right)(10)\right] - \left[9.5 + \left(\frac{8-4}{6}\right)(10)\right]$$

$= 34.5 - 16.167$

$= 18.333$

Quite a number of candidates used the position of the quartiles as the values of the quartiles. Some used the number of students and not the score as the values of the quartiles. They did not know that the formula for median can be used to find the first and third quartiles. However, most of them knew that the interquartile range is $Q_3 - Q_1$.

(i)

b) $Q_3 = \frac{3}{4} \times 32$ P0 K

$= 24$ P0

$Q_1 = \frac{1}{4} \times 32$

$= 8$

Interquartile range $= Q_3 - Q_1$

$= 16.$ N0

(ii) $\frac{3}{4} \times 32 = 24$

$\frac{1}{4} \times 32 = 8$

$\therefore$ interquartile range $= 24 - 8$

$= 16$

$\therefore \ = 26.5$ #

Among the few that used the formula for median, some did not know the meaning of the symbols, such as L, F and f_m , that were being used. Hence, the candidates failed to determine the first and third quartiles.The size of class was also incorrect. Below are the examples of mistakes made by the candidates:

(i) Both the size of class and the values of *L* were wrong.

first quartile $= 20.5 + \left(\frac{\frac{1}{4}(32) - 10}{10} \right) 9$

$= 20.5 + (-1.8)$

$= \cancel{22.3}\ 18.7$

F

third quartile $= 20.5 + \left(\frac{\frac{3}{4}(32) - 10}{10} \right) 9$

$= 20.5 + 12.6$

$= 33.1$

(ii) Candidates used the median class,ie 20–29 when finding the first and third quartiles.

$Q_1 = L + \left(\frac{\frac{1}{4}N - F}{fm} \right) c$

$= 19.5 + \left(\frac{8 - 10}{10} \right) 10$

$= 17.5$

$Q_3 = 19.5 + \left(\frac{24 - 10}{10} \right) 10$

$= 33.5$

Interquartile range $= 33.5 - 17.5$

$= 16$

(iii) Values of F and f_m interchanged.

$$Q_1 = 9.5 + \left(\frac{\frac{1}{4}(32) - 6}{4}\right)10 \qquad Q_3 = 29.5 + \left(\frac{\frac{3}{4}(32) - 8}{10}\right)10$$

(iv) Candidates did not understand the meaning of interquartile range. Candidates took the median as the interquartile range.

$$\begin{aligned}\text{Interquartile range} &= Q_3 - Q_1\\ &= Q_2\\ &= \text{median}\end{aligned}$$

$$\begin{aligned}\text{Median, } m &= L + \left(\frac{\frac{1}{2}N - F}{fm}\right)c\\ &= 9.5 + \left(\frac{\frac{1}{2}(32) - 10}{10}\right)10\\ &= 9.5 + \left(\frac{16-10}{10}\right)10\\ &= 9.5 + 6\\ &= 15.5\end{aligned}$$

(v) Candidates used the same class for both the quartiles.

$$\begin{aligned}Q_1 &= L + \left(\frac{\frac{1}{4}N - F}{fm}\right)c\\ &= 19.5 + \left(\frac{8 - 10}{10}\right)10\end{aligned}$$

$$Q_3 = 19.5 + \left(\frac{24 - 10}{10}\right)10$$

Some candidates used the wrong formula.

(i) $\text{3rd quartile} = 29.5\left(\frac{\frac{3}{4}(32) - 20}{8}\right)10 = 147.5$ $\qquad$ $\text{1st quartile} = 9.5\left(\frac{\frac{1}{4}(32) - 4}{6}\right)10 = 63\frac{1}{3}$

(ii)

$$= L\left(\frac{\frac{3}{4}(32)-10}{8}\right)10 - L\left(\frac{\frac{1}{4}(32)-4}{6}\right)10$$

$$29.5\left(\frac{24-10}{8}\right)10 - 9.5\left(\frac{8-4}{6}\right)10$$

(iii)

$$3^{rd}\left(\frac{L+\frac{3}{4}N-F}{fm}\right)c - 1^{st}\left(\frac{L+\frac{1}{4}N-F}{fm}\right)c$$

P1

$$10\left(\frac{29.5+\frac{3}{4}(32)-10}{20}\right) - 10\left(\frac{19.5+\frac{1}{4}(32)-6}{10}\right)$$

(iv)

$$Q_1 = L + \left(\frac{\frac{1}{4}N + F}{fm}\right)c$$

P1

$$= 9.5 + \left(\frac{8+4}{6}\right)10$$

Question 6(a)

Most candidates could answer part (a). Candidates were able to find the number of the top most block, *n*, by using simple calculation, that is $n = \frac{300}{15}$. Candidates could also use the formula $T_n = a + (n-1)d$ and substitute the corresponding values correctly to find the length of the top most block. Hence, the candidates were able to use the formula $S_n = \frac{n}{2}\left[2a + (n-1)d\right]$ and substitute the corresponding values correctly to find the total length of the blocks.

(i) 3m = 300 cm

$$\frac{300}{15} = 20 \text{ blocks}$$

$$T_{20} = a + 19d$$
$$= 985 + 19(-30)$$
$$= 415 \text{ cm}$$

(ii)

$$S_{20} = \frac{20}{2}\left[2(985) + 19(-30)\right]$$
$$= 10(1400)$$
$$= 14000 \text{ cm}$$

Some candidates listed out all the terms to get the answers.

985, 955, 925, 895, 865, 835, 805, 775, 745, 715, 685, 655, 625, 595, 565, 535, 505, 475, 445, 415

top most block

00 cm ÷ 15 cm = 20 block

The length of top most block = 415 cm N1

(ii) Total length of the blocks = 985 + 955 + 925 + 895 + 865 + 835 + 805 + 775 + 745 + 715 + 685 + 655 + 625 + 595 + 565 + 535 + 505 + 475 + 445 + ... = 14000 N1

There were some candidates who could not find the number of stairs, *n*, although it involved only a simple calculation. The candidates just simply assumed the value of *n* as 30 or 21. However, the candidates knew how to apply the formula of $T_n = a + (n-1)d$ and $S_n = \frac{n}{2}(2a+(n-1)d$, to solve the problems.

(i)

T_{30} = 985 + (-1)(-30)

= 115 cm.

S_{30} = $\frac{30}{2}$ [2(985) + 29(-30)

S_{30} = 15 [1970 + (-870)]

= 15 (1100)

= 16500 cm.

(ii)

$T_{21} = 985 + (21-1)\ 30$
$= 985 + 20(30)$
$= 985 + 600$
$T_{21} = 385\ cm$ //

$S_n = \frac{n}{2}[a + l]$
$S_{21} = \frac{21}{2}(985 + 385)$
$= \frac{21}{2}(1370)$
$= 14385\ cm$ //

There were candidates who used the common difference, d = 30 (positive) for a = 985cm.

(i)

i) $T_{20} = 985 - 19(30)$

(ii)

$S_{20} = \frac{20}{2}[2(985) + (20-1)30]$
$= 10(1970 + 570)$
$= 25400cm$ //

Question 6(b)

Candidates could find the maximum number of the stairs, n, and then multiplied it by 15 to get the maximum height of the stairs. The candidates could use various methods to get the value of n. Below are the examples of the various methods used by the candidates.

(i) Candidates used T_n = 0 and were able to choose the suitable value of n.

(b) $T_n = 0$ or negative

$T_n = 985 + (n-1) -30 = 0$ K1
$985 - 30n + 30 = 0$
$985 + 30 = 30n$
$30n = 1015$
$n = 33.83$ N1

$T_{33} = 985 + 32(-30)$
$= 25\ cm$

T_{34} = negative value
$= 985 + (33(-30))$
$= -5$

length does not exist in negative.

T_{33} is the top most stair
so, the maximum height
is 33×15 cm
$= 495$ cm or N1
4.95 m.

(ii) Candidates used $T_n < 0$ or $T_n > 0$ and were able to identify the correct value of *n.*

a)

(b) $T_n < 0$

$985 + (n-1)(-30) < 0$ ✓ M1

$985 - 30n + 30 < 0$

$30n > 1015$

$n > 33.83$ ✓ N1

∴ maximum blocks are 33 ✓

∴ maximum height of stairs

$= 33 \times 15$ cm

$= 495$ cm ✓ N1

b)

$T_n > 0$

$985 - (n-1)30 > 0$

$985 - 30n + 30 > 0$

$-30n > -1015$

$n < 33.8$

$n = 33$

33×15 cm $= 495$ cm

$= 4.95$ m

(iii) Candidates used $T_n = 30$ and were able to identify the value of *n*.

$T_n = 985 + (n-1)(-30) = 30$ ✓

$985 - 30n + 30 = 30$

$-30n = -985$

$30n = 985$

$n = 32.8$

$n = 33$

max height $= 33 \times 15$

$= \underline{\underline{495 \text{ cm}}}$

Below are the mistakes made by the candidates in finding the maximum height of the stairs;

(i) There were quite a number of candidates who just used $n = 20$ for the height of 3 m from part (a) to find the maximum height of the stairs.

6 c) 15×20

$= \underline{300 \text{ cm}}$

(ii) There were a few candidates who used the formula for the sum to infinity of the geometric progression to find the maximum height of the stairs.

$$S_\infty = \frac{(985)}{1-30}$$

$$= 33.79$$

(iii) Candidates did not use integer for the value of n in their calculations whereas n refers to the number of stairs.

a)

$33.8333 \times 15 = 507.5$ cm or 5.075 m

b)

(b) $T_n = 985 + (n-1)(-30) = 0$

$= 985 - 30n + 30 = 0$

$-30n = 1015$

$n = 33.83$

$33.83 \times 15 = 507.5$ cm

$= 5.075$ m

Question 7

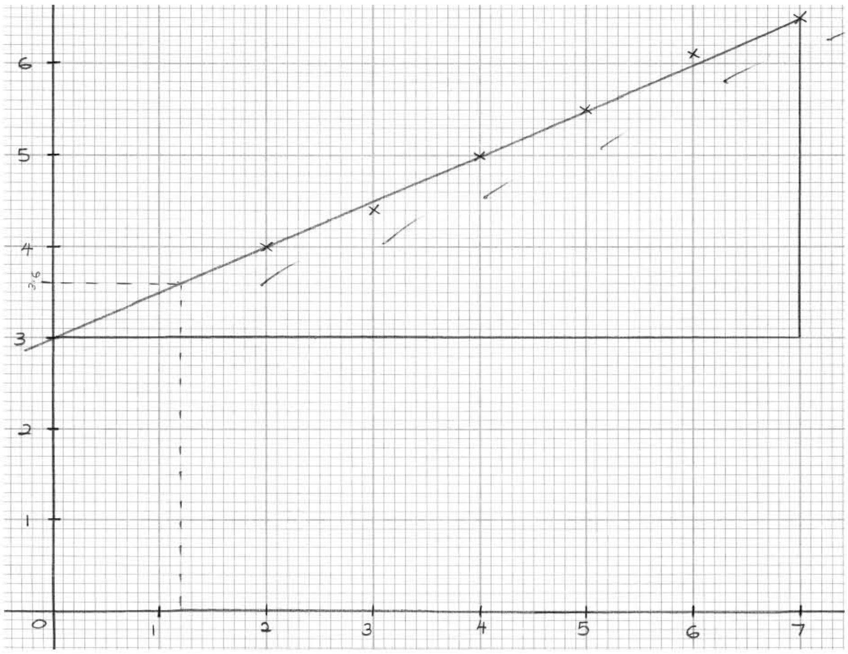
All the candidates who answered this question could get the values of $\frac{y}{x}$ easily. Almost all knew how to plot the points accurately from the table and draw the graph of $\frac{y}{x}$ against x according to the given scales. They also had no problem in drawing the line of best fit. Candidates managed to reduce the non-linear equation into linear form, $Y = mX + c$. They knew how to find the gradient of the line from the graph using the gradient formula and were also able to read the Y-intercept. They could then equate the gradient to $2k$ and the Y-intercept to $\frac{p}{k}$, hence finding the values of p and k. They could read the value of $\frac{y}{x}$ from the graph when $x = 1.2$, then multiplied by 1.2 to get the value of y.

x	2	3	4	5	6	7
y/x	4.0	4.4	5.0	5.5	6.1	6.5

$$y = 2kx^2 + \frac{p}{k}x$$

$$\frac{y}{x} = 2kx + \frac{p}{k}$$

Reducing non-linear equation to linear form
$Y = mX + c$



$$2k = \frac{6.5 - 4.0}{7 - 2}$$

$$2k = \frac{2.5}{5}$$

$$2k = 0.5$$

$$k = \frac{0.5}{2}$$

$$k = 0.25 \quad N1$$

$$\frac{p}{k} = 3$$ ...substitute $k = 0.25$ K1

$$p = 3(0.25)$$

$$p = 0.75 \quad N1$$

(iii) when $X = 1.2$

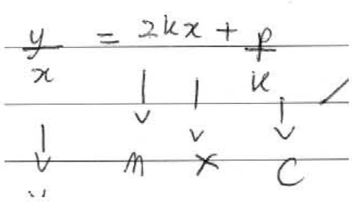
$$y/x = 3.6$$

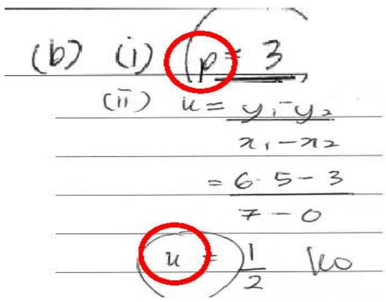
$$\frac{y}{x} = 3.6$$

$$\frac{y}{1.2} = 3.6$$

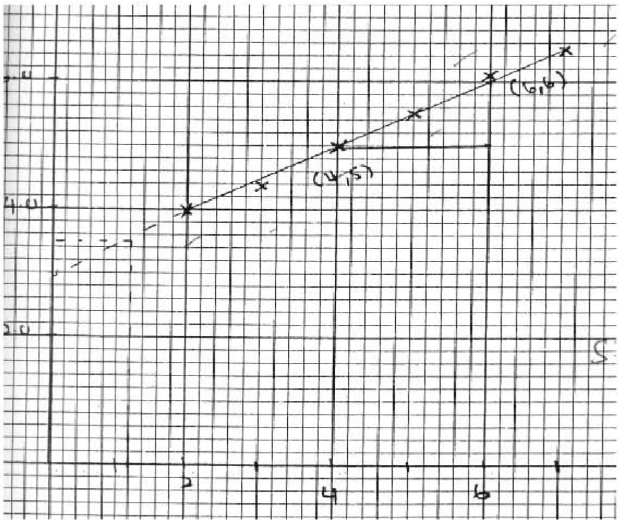
$$y = 4.32$$

Some candidates were able to change to linear form but could not relate the gradient and the Y-intercept to the constants, $2k$ and $\frac{p}{k}$ respectively.

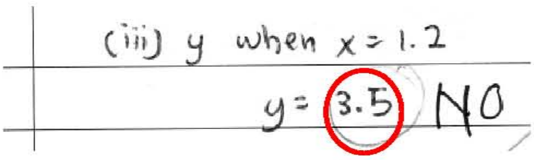




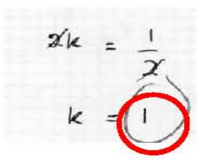
A few of the candidates did not use the given scales when plotting the graph.



Quite a number of candidates failed to answer (b)(iii). They took the value of $\frac{y}{x}$ from the graph as their answer to y.



Some candidates were still having problem in solving a simple equation.



A few candidates did not answer the question according to the instructions as required by the question. The candidates did not read the *Y*- intercept or find the gradient from the graph but they substituted the points into *Y* = *mX* + *c* to find the value of *k* and *p*.

(i)

$$(4) = 2k(2) + 3$$
$$1 = 4k$$
$$k = \frac{1}{4}$$

(ii)

$$y = mx + c$$
$$6 = \frac{1}{2}(\cancel{6}^{\,3}) + c$$
$$6 = 3 + c$$
$$c = 3$$
$$\frac{y}{x} = \frac{1}{2}x + 3$$

Question 8(a)

The majority of the candidates were able to answer only this part of the question correctly. Candidates were able to use the triangle law correctly.

(a)(i) $\overrightarrow{BP} = \overrightarrow{BO} + \overrightarrow{OP}$ K1

$= -6\underset{\sim}{y} + \frac{1}{4}(\overrightarrow{OA})$

$= -6\underset{\sim}{y} + \frac{1}{4}(8\underset{\sim}{x})$

$= -6\underset{\sim}{y} + 2\underset{\sim}{x}$ M1

(ii) $\overrightarrow{OQ} = \overrightarrow{OA} + \frac{1}{2}(\overrightarrow{AB})$

$= 8\underset{\sim}{x} + \frac{1}{2}(-8\underset{\sim}{x} + 6\underset{\sim}{y})$

$= 8\underset{\sim}{x} - 4\underset{\sim}{x} + 3\underset{\sim}{y}$

$= 4\underset{\sim}{x} + 3\underset{\sim}{y}$ M1

However, there were still a few candidates who were not able to use the triangle law to find $\overrightarrow{BP}$ or $\overrightarrow{OQ}$ correctly.

(i)

(a)

i) $\overrightarrow{BP} = \overrightarrow{OB} + \overrightarrow{OP}$ K0

$= 6\underset{\sim}{y} + 2\underset{\sim}{x}$ N0

ii) $\overrightarrow{OQ} = \overrightarrow{OB} + \overrightarrow{OA}$

$= 6\underset{\sim}{y} + 8\underset{\sim}{x}$ N0

(ii)

(i) $\overrightarrow{BP} = \overrightarrow{PO} + \overrightarrow{OB}$

$= -\underset{\sim}{x} + 6\underset{\sim}{y}$

Some candidates could not interpret *OA : OP* = 4 : 1 and *AB : AQ* = 2 : 1 correctly to find $\overrightarrow{BP}$ or $\overrightarrow{OQ}$ as seen in the examples below.

(i)

8. a) $\overrightarrow{OA} = 8\underset{\sim}{x}$ $\qquad \overrightarrow{OP} = \left(\frac{1}{5}\right) \times 8(\underset{\sim}{x})$

i) $\overrightarrow{BP} = \overrightarrow{BO} + \overrightarrow{OP}$ ✓ K1 $\qquad = \frac{8}{5}\underset{\sim}{x}$

$= -6\underset{\sim}{y} + \frac{8}{5}\underset{\sim}{x}$ N0

$\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -8\underset{\sim}{x} + 6\underset{\sim}{y}$

ii) $\overrightarrow{OQ} = \overrightarrow{OA} + \overrightarrow{AQ}$

$= -6\underset{\sim}{y} + \frac{8}{5}\underset{\sim}{x} + \left(-\frac{8}{3}\underset{\sim}{x} + 2\underset{\sim}{y}\right)$ $\qquad AQ = \left(\frac{1}{3}\right) \times (-8\underset{\sim}{x} + 6\underset{\sim}{y})$

$= -4\underset{\sim}{y} - \frac{16}{15}\underset{\sim}{x}$ N0 $\qquad = -\frac{8}{3}\underset{\sim}{x} + 2\underset{\sim}{y}$

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(ii)

8 a) i- $\overrightarrow{BP} = \overrightarrow{BO} + \overrightarrow{OP}$ ✓ K1

$= -6\underset{\sim}{y} + \frac{8}{5}\underset{\sim}{x}$ N0

ii- $\overrightarrow{OQ} = \overrightarrow{OA} + \overrightarrow{AQ}$ ✓

$= 8\underset{\sim}{x} + \frac{10}{3}$ N0

(iii)

(8) (a) (i) $\overrightarrow{BP} = \overrightarrow{BO} + \overrightarrow{OP}$ ✓ K1 $\qquad \overrightarrow{OP} = \left(\frac{1}{5}\right)(\overrightarrow{OA})$

$= -6\underset{\sim}{y} + \frac{8}{5}\underset{\sim}{x}$ N0 $\qquad = \frac{1}{5}(8\underset{\sim}{x})$

$= \frac{8}{5}\underset{\sim}{x}$.

(iv)

(ii) $\overrightarrow{AQ} = \left(\frac{1}{3}\right)(\overrightarrow{AO} + \overrightarrow{OB})$

$= \frac{1}{3}(-8\underset{\sim}{x} + 6\underset{\sim}{y})$

$= \left(-\frac{8}{3}\underset{\sim}{x} + 2\underset{\sim}{y}\right)$

$\overrightarrow{OQ} = \overrightarrow{OA} + \overrightarrow{AQ}$ ✓

$= 8\underset{\sim}{x} + \left(-\frac{8}{3}\underset{\sim}{x} + 2\underset{\sim}{y}\right)$

$= 8\underset{\sim}{x} - \frac{8}{3}\underset{\sim}{x} + 2\underset{\sim}{y}$

$= \frac{16}{3}\underset{\sim}{x} + 2\underset{\sim}{y}$ N0

(v)

$$\text{(ii)}\quad \overrightarrow{OQ} = \overrightarrow{OA} + \overrightarrow{AQ} \quad \checkmark$$
$$= 8\underset{\sim}{x} + 4\underset{\sim}{y}$$

A few candidates did not understand the concept of negative vector, $\overrightarrow{BO} = -\overrightarrow{OB}$.

$\overrightarrow{OB} = 6\underline{y}$,
$\therefore \overrightarrow{BO} = -6\underline{y}$

$$\overrightarrow{BP} = \overrightarrow{BO} + \overrightarrow{OP} \quad \checkmark \; K1$$
$$= 6\underset{\sim}{y} + \overrightarrow{OP} \rightsquigarrow 8\underset{\sim}{x} \div 4 = 2\underset{\sim}{x}$$
$$\overrightarrow{BP} = 6\underset{\sim}{y} + 2\underset{\sim}{x} \qquad M0$$

Candidates were careless when dealing with operations in algebra.

$8x - 4x = 4x$

$$\text{ii)}\quad \overrightarrow{OQ} = \overrightarrow{OA} + \overrightarrow{AQ}$$
$$= 8\underset{\sim}{x} + \tfrac{1}{2}(-8\underset{\sim}{x} + 6\underset{\sim}{y})$$
$$= 8\underset{\sim}{x} + (-4\underset{\sim}{x} + 3\underset{\sim}{y}) \quad \checkmark$$
$$= 3\underset{\sim}{x} + 3\underset{\sim}{y} \qquad M0$$

Question 8(b)

Candidates in the high achievement group could relate $\overrightarrow{OS} = h\overrightarrow{OQ}$ to $\overrightarrow{OS} = \overrightarrow{OB} + k\overrightarrow{BP}$ or equivalent. They could equate the coefficients of $\underline{x}$ and $\underline{y}$, then solve simultaneously to find the values of *h* and *k*.

$$\text{b)}\quad \overrightarrow{OS} = h\,\overrightarrow{OQ} \quad \text{and} \quad \overrightarrow{BS} = k\,\overrightarrow{BP}$$
$$\overrightarrow{OS} = h(4\underset{\sim}{x} + 3\underset{\sim}{y}) \qquad\qquad \overrightarrow{BS} = k(2\underset{\sim}{x} - 6\underset{\sim}{y})$$
$$= 4h\underset{\sim}{x} + 3h\underset{\sim}{y} \quad \checkmark \; P1 \qquad\qquad = 2k\underset{\sim}{x} - 6k\underset{\sim}{y} \quad \checkmark$$

$$\overrightarrow{OS} = \overrightarrow{OB} + \overrightarrow{BS}$$
$$4h\underset{\sim}{x} + 3h\underset{\sim}{y} = 6\underset{\sim}{y} + 2k\underset{\sim}{x} - 6k\underset{\sim}{y} \quad \checkmark \; K1$$
$$4h\underset{\sim}{x} + 3h\underset{\sim}{y} = 2k\underset{\sim}{x} + (6 - 6k)\underset{\sim}{y}$$

By comparison,

$$4h = 2k \quad \checkmark \qquad\qquad 3h = (6 - 6k) \quad \checkmark$$
$$\cancel{\tfrac{4}{2} = \tfrac{k}{h}} \qquad\qquad 3h = 6 - 6k$$
$$2h = k \; \text{—①} \qquad\qquad h = 2 - 2k \; \text{—②}$$

substitute ① in ② substitute $h = 2/5$ in ①

$h = 2 - 2(2h)$ ✓ M1 $2\left(\frac{2}{5}\right) = k$

$h = 2 - 4h$ $k = \frac{4}{5}$ ✓ M1

$5h = 2$

$h = \frac{2}{5}$ ✓ M1

Some candidates could not use the relationship, $\overrightarrow{OS} = \overrightarrow{OB} + \overrightarrow{BS}$ to find the value of *h* and of*k.* Most of the candidates used $\overrightarrow{OS} = \overrightarrow{BS}$. Examples are shown below:

(i)

b) $\overrightarrow{OS} = h\,\overrightarrow{OQ}$ P1 $\overrightarrow{BS} = k\,\overrightarrow{BP}$

$\overrightarrow{OS} = h(4\underset{\sim}{x} + 3\underset{\sim}{y})$ ✓ $= k(-6y + 2x)$

$h = \frac{1}{4}, 4$ $k = \frac{1}{2}, 2$

(ii)

b) $\overrightarrow{OS} = 4h\underset{\sim}{x} + \cancel{3y}\ 3h\underset{\sim}{y}$ ✓ P1

$\overrightarrow{BS} = \cancel{\ } 2k\underset{\sim}{x} - 6k\underset{\sim}{x}$

$4h - 2k = 0$ $3h + 6k = 0$ K0

$4h = 2k$ $k = 2h,\ 3h + 12h = 0$ K0

$k = 2h$. M0 $h = 0$ M0

Tulis di kedua dua belah muka surat

(iii)

$\overrightarrow{OS} = h\,\overrightarrow{OQ}$ P1 $\overrightarrow{BS} = k\,\overrightarrow{BP}$

$\overrightarrow{OS} = h(6\underset{\sim}{y} + 8\underset{\sim}{x})$ ✓ $\overrightarrow{BS} = k(2\underset{\sim}{x} - 6\underset{\sim}{y})$ ✓

$\overrightarrow{OS} = 6h\underset{\sim}{y} + 8h\underset{\sim}{x}$ —① $\overrightarrow{BS} = 2\underset{\sim}{x}k - 6\underset{\sim}{y}k$ —②

Compare ① and ②

$6h\underset{\sim}{y} + \boxed{8h\underset{\sim}{x}} = -6k\underset{\sim}{y} + \boxed{2k\underset{\sim}{x}}$ K0 K0

(iv)

$\overrightarrow{OS} = m\overrightarrow{BS}$

$3h\underline{y} + 4h\underline{x} = m(2k\underline{x} - 6k\underline{y})$

$3h\underline{y} + 4h\underline{x} = 2km\underline{x} - 6km\underline{y}$

(v)

b) $\overrightarrow{OS} = \overrightarrow{OB} + \overrightarrow{BS}$

$= 6\underset{\sim}{y} + k\overrightarrow{BP}$ ✓ K1

$= 6\underset{\sim}{y} + k(2\underset{\sim}{x} - 6\underset{\sim}{y})$ ✓ P1

$= 6\underset{\sim}{y} + 2k\underset{\sim}{x} - 6k\underset{\sim}{y}$

$\therefore h = \frac{1}{3}$ K0 N0

$k = \frac{5}{6}$ N0

(vi)

$\overrightarrow{OS} = h\overrightarrow{OQ}$

$(\underset{\sim}{x} + \underset{\sim}{y}) = h(4\underset{\sim}{x} + 3\underset{\sim}{y})$ ✓ P1

$(\underset{\sim}{x} + \underset{\sim}{y}) = 4h\underset{\sim}{x} + 3h\underset{\sim}{y}$

$\overrightarrow{BS} = k\overrightarrow{BP}$

$(\underset{\sim}{x} + \underset{\sim}{y}) = k(2\underset{\sim}{x} - 6\underset{\sim}{y})$ ✗

$(\underset{\sim}{x} + \underset{\sim}{y}) = 2k\underset{\sim}{x} - 6k\underset{\sim}{y}$

equating weficient x

$4h = 1$

$h = 1/4$ # N0

K0 K0

~~3h = 1~~

~~h = 1/3~~

equating ceeficient x

$2k = 1$

$k = 1/2$ # N0

(vii)

b) $\overrightarrow{OS} = h\overrightarrow{OQ}$

$= h(12\underset{\sim}{x} + 3\underset{\sim}{y})$ ✓ P1

$= 3(4\underset{\sim}{x} + \underset{\sim}{y})$

$\therefore h = 3$ #

K0 K0 N0

$\overrightarrow{BS} = k\overrightarrow{BP}$

$= k(-6\underset{\sim}{y} + 2\underset{\sim}{x})$

$= 2(-3\underset{\sim}{y} + \underset{\sim}{x})$

$\therefore k = 2$ #

N0

There were a few candidates who tried to solve only a small part of the question.

(i)

b) $\vec{OS} = h\vec{OQ}$ $\quad\quad \vec{BS} = k\vec{BP}$

$= h(4\underset{\sim}{x} + 3\underset{\sim}{y}) \quad\quad = k(2\underset{\sim}{x} - 6\underset{\sim}{y})$ P1

$= 4h\underset{\sim}{x} + 3h\underset{\sim}{y} \quad\quad = 2k\underset{\sim}{x} - 6k\underset{\sim}{y}$

(ii)

$\vec{BS} = k\vec{BP}$

$\vec{BO} + \vec{OS} = k(\vec{BO} + \vec{OP})$

Some candidates failed to give the value of *h* or *k* in the simplest fraction.

$\vec{OS} = h(\vec{OQ})$

$= h(4\underset{\sim}{x} + 3\underset{\sim}{y})$ P1

$= 4h\underset{\sim}{x} + 3h\underset{\sim}{y}$

$\vec{OS} = \vec{OB} + \vec{BS}$

$= 6\underset{\sim}{y} + k\vec{BP}$ K1

$= 6\underset{\sim}{y} + k(-6\underset{\sim}{y} + 2\underset{\sim}{x})$

$= 6\underset{\sim}{y} - 6k\underset{\sim}{y} + 2k\underset{\sim}{x}$

$\underset{\sim}{x} + 3h\underset{\sim}{y} = (6\underset{\sim}{y} - 6k\underset{\sim}{y}) + 2k\underset{\sim}{x}$

$4h = 2k \quad\quad 3h = 6 - 6k$

$h = \frac{2k}{4} \quad\quad 3(\frac{1}{2}k) = 6 - 6k$ K1

$h = \frac{1}{2}k \quad\quad \frac{3}{2}k = 6 - 6k$

$\frac{3}{2}k + 6k = 6$

$h = \frac{1}{2}\left(\frac{12}{15}\right) \quad\quad \frac{15}{2}k = 6$

$= \frac{2}{5}$ M1 $\quad\quad 15k = 12$

$k = \frac{12}{15}$ $\boxed{\text{N0}}$

Question 8(c)

Candidates in the high achievement group were able to use Pythagoras Theorem with $|\underset{\sim}{x}| = 2$ and $|\underset{\sim}{y}| = 3$ to find magnitude of $\vec{AB}$.

$\vec{AB} = \sqrt{16^2 + 18^2}$ K1

$= \sqrt{580}$ N1

$= 24.083$ unit

However many candidates did not use the Pythagoras Theorem to find the magnitude of $\overrightarrow{AB}$ as shown in the examples below.

(i)

c) $|\overrightarrow{AB}| = \cancel{OA}\ \overrightarrow{AO} + \overrightarrow{OB}$ $\quad x = 2 \quad y = 3$

$= -8\underset{\sim}{x} + 6\underset{\sim}{y}$

$= -8(2) + 6(3)$ KO

$= 2$ units NO

(ii)

c) $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB}$

$= (8\underset{\sim}{x} + 6\underset{\sim}{y})$

$|\overrightarrow{AB}| = 8\underset{\sim}{x} + 6\underset{\sim}{y}$

$= 8|\underset{\sim}{x}| + 6|\underset{\sim}{y}|$

$= 8[2] + 6[3]$ KO

$= 16 + 18$

$= 34$ units NO

(iii)

c) $\overrightarrow{AB} = -8\underset{\sim}{x} + 6\underset{\sim}{y}$

$|\overrightarrow{AB}| = \dfrac{-2\underset{\sim}{x}}{2} + \dfrac{6\underset{\sim}{y}}{3}$

$= -\underset{\sim}{x} + 2\underset{\sim}{y}$

Most candidates could not apply $|\underset{\sim}{x}| = 2$ and $|\underset{\sim}{y}| = 3$ into the Pythagoras Theorem correctly to find $|\overrightarrow{AB}|$.

(i)

$$(c)\ |\overrightarrow{AB}| = \sqrt{6y^2+8x^2}$$
$$= \sqrt{6(3)^2+8(2)^2}$$
$$= \sqrt{54+32}$$
$$= \sqrt{86}$$
$$= 9.2736$$

(ii)

$$AB^2 = 3^2+2^2$$
$$= 13$$
$$AB = \sqrt{13}$$
$$= 3.605$$

(iii)

$$|\overrightarrow{AB}| = |\overrightarrow{AO}| + |\overrightarrow{OB}|$$
$$= \sqrt{-8(2)^2+6(3)^2}$$
$$= \sqrt{-32+54}$$
$$= \sqrt{22}$$
$$= \underline{4.69} \text{ unit}$$

(iv)

$$\overrightarrow{AB} = \sqrt{8(2)+6(3)}$$
$$= \sqrt{16+18}$$
$$= \sqrt{34}$$

A few candidates interpreted magnitude of a vector, $|\overrightarrow{AB}|$ as a unit vector which was incorrect.

$$|\overrightarrow{AB}| = \frac{AB}{|AB|} \qquad \overrightarrow{AB} = AO+OB = -8x+6y$$
$$= \frac{-8x+6y}{\sqrt{(8)^2+(6)^2}}$$
$$= \frac{-8x+6y}{\sqrt{100}}$$
$$= \frac{-8x+6y}{10}$$
$$= \frac{1}{10}\begin{pmatrix}-8(2)\\6(3)\end{pmatrix}$$
$$= \frac{1}{10}\begin{pmatrix}-16\\18\end{pmatrix} \quad \text{NO}$$

However, a few candidates gave the answer to one decimal place whereas the answer should be given to at least to two decimal places.

$$= \sqrt{(6(3))^2+(8(2))^2}$$
$$= 24.1$$

Question 9

On the whole, quite a number of candidates who answered this question, could use the formula $s = r\theta$ but had the problem of finding the radius of the sector *APB*. Not many candidates could answer part (b) but some candidates managed to find the area of the shaded region through various ways.

The candidates who had answered this question, could change 60° into radian but only a few of them could find the length of *OP* using the trigonometric ratio and then used *OP* + 10 cm to find the radius of the sector *APB*. They could use the formula $s = r\theta$ correctly to find the length of the arc *AB*. Candidates could also use the formula $A = ½r^2\theta$ and manage to find the area of the shaded region. However, there were some candidates who used angles in degrees.

9 a) $60° = 60 \times \frac{3.142}{180}$

$= 1.0473$ rad ✓ P1

$AB = 1.0473 \times 30$ — K1

$= 31.419$ cm ✓ N1

$\frac{OP}{\sin 60} = \frac{10}{\sin 30°}$

$OP = 17.32$ cm

$OP^2 = 10^2 + 17.32^2$

$= 399.98$

$OP = 19.9995$

$= 20$ cm — K1

$AP = 20 + 10$

$= 30$ ✓ K1

b) $\frac{1}{2} r^2 \theta$

$\frac{1}{2}(30^2)(1.0473) = 471.29 \text{ cm}^2$ ✓ K1

$\frac{1}{2} \times 10 \times 17.32 = 86.6 \times 2$ ✓

$= 173.2 \text{ cm}^2$ K1

$240° = 4.1893$ rad — P1

$\frac{1}{2} \times 4.1893 \times 10^2 = 209.465$

$= 471.29 - 173.2 - 209.47$ — K1

$= 88.62 \text{ cm}^2$ ✓ N1

Q9

10

Some of the candidates could get the value of the radius of the sector *APB* correctly, that was 30cm, without showing any calculation, in the formula $s = r\theta$ to find the length of the arc *AB*.

9. a) $r = r\theta$. $60° = \frac{60 \times 3.142}{180}$

$= 1.0473$ rad

$AB = 30(1.0473)$

$= 31.419$ cm.

Quite a number of candidates failed to find the radius of sector *APB* correctly. For examples:

(i) The candidates assumed *AQ* = *BR* = 10 cm.

(a)

9. (a) the length of Arc AB = $r\theta$

length of QP = 17.32

$\tan 60 = \frac{QP}{QO}$

$\tan 60 = \frac{QP}{10}$ KO

$\tan 60 \times 10 = QP$

$17.32 = QP$ KO

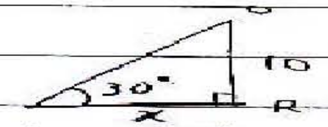
AQ = 10 cm $r = 10 + 17.32 = 27.32$

arc AB $= 27.32\left(\frac{60}{180} \times \frac{22}{7}\right)$ K1

$= 27.32\,(1.0472)$

$= 28.6$ cm NO

(b)



$\tan 30° = \frac{10}{x}$

$x = \frac{10}{\tan 30°}$

$= 17.32$

radius PB =

$10 + 17.32$

(ii) The candidates assumed points *Q* and *R* were midpoints of *PA* and *PB* respectively.

a) Length of arc AB = $r\theta$

$s = r\theta$

$s = (17.32 \times 2)(1.047)$

$= 36.27\text{cm}$

(iii) Candidates simply used any value as the radius of sector *APB* such as 10 cm or 20 cm.

(a)

$\theta = 1.04733$ ✓

$S = (10 \times) 1.04733$

$= 10.4733 \text{ cm}$

(b)

$S = r\theta$

$S = (20) \times 1.047$

$= 20.94 \; \cancel{cm^2} \; cm$

Some candidates were not able to use the correct method to find the area of the shaded region. For example, candidates used the area of the shaded region = (area of sector *APB* – area of circle inscribed in the sector *APB*).

(i)

(b) Area of shaded region

$= \frac{1}{2} r^2\theta - \pi r^2$

$= \left(\frac{1}{2} \times 20^2 \times 1.047\right) - (3.142 \times 10^2)$

(ii)

area of shaded region $= 471.24 - 314.1593$

$= 157.081 \text{ cm}^2$

Question 10

This question was not popular among the candidates. Only candidates in the high achievement group managed to answer this question well.

The candidates could find the value of *k* by differentiating $y = k(x-1)^3$, substituting $x = 3$ and equate to 24.

For the question 10b (i), candidates managed to get one of the integration limits by taking $y = 0$ and solved $2(x - 1)^3 = 0$. Candidates were able to find the area of the shaded region, *P*, by integrating $\int_1^3 2(x-1)^3 \, dx$.

In question 10b (ii), candidates could find the volume of revolution when the region *R* is revolved through $360°$ about the *x*-axis by integrating $\int_0^1 \pi [2(x-1)^3]^2 \ dx$.

$y = k(x-1)^3$

$\frac{dy}{dx} = 3k(x-1)^2 \times 1$

$= 3k(x-1)^2$

$\frac{dy}{dx} = 24$, when $x = 3$

$3k(3-1)^2 = 24$

$3k(4) = 24$

$12k = 24$

$k = 2$

(i) $\int_1^3 2(x-1)^3 \ dx$ P1

$\left[\frac{2(x-1)^4}{4}\right]_1^3$ K1

$= \left[\frac{2(3-1)^4}{4} - \frac{2(1-1)^4}{4}\right]$

$= 8$ unit^2 N1

$\pi \int_0^1 y^2 \ dx$

$\pi \int_0^1 (2(x-1)^3)^2 \ dx$

$\pi \int_0^1 4(x-1)^6 \ dx$

$\pi \left[\frac{4(x-1)^7}{7}\right]_0^1$ K1

$\pi \left[\frac{4(1-1)^7}{7} - \frac{4(0-1)^7}{7}\right]$

$= \pi \left[0 - \left(-\frac{4}{7}\right)\right]$

$= \frac{4}{7}\pi$ unit^3 N1

However, there were a few candidates who were not able to find the value of *k*. They used the wrong method such as $y = k(x-1)^3 = \frac{dy}{dx}$ whereas $y \neq \frac{dy}{dx}$

(i)

$\frac{dy}{dx} = k(x-1)^3$

$= 2k(x-1)^2$

$= 2k(2)^2$

$24 = 8k$

(ii)

$\frac{dy}{dx} = 24$

$y = \int 24$

$= 24x$ K0

$y = k(x-1)^3$

$24x = k(3-1)^3$

$24(3) = 8k$ K0

$k = \frac{72}{8} = 9$

Candidates were weak in algebraic manipulation such as simplifying $9(x-1)^3 = 0$ as shown in (i) and expanding $2(x-1)^3$ as shown in (ii) below.

(i)

$$0 = 9(x-1)^3$$
$$= (9x-9)^3$$
$$0 = 9x-9$$
$$x = 1$$

(ii)

$$\text{So}, \int_1^3 2x^3 - 6x - 2$$
$$= \left[\frac{2x^4}{4} - \frac{6x^2}{2} - 2x\right]_1^3$$

There were a few candidates who did not know how to integrate the function in the form of $k(ax+b)^n$.

(i)

$$= \int_1^3 2(x-1)^3\,dx$$
$$= 6(x-1)^2\Big|_1^3$$

(ii)

$$\pi\int_1^3 4(x-1)^6$$
$$= \frac{4x(x-1)^7}{7(1)}\Bigg|_1^3$$

(iii)

$$= \int_1^3 9(x-1)^3\,dx$$
$$= [27(x-1)^2]_1^3$$

(iv)

$$= \pi\int_1^3 81(x-1)^6$$
$$= \pi[486(x-1)^5]_1^3$$

For question 10b (ii), candidates were weak in applying the law of indices, $(x^m)^n = x^{mn}$. For example, they applied $[(x-1)^3]^2$ as $(x-1)^5$.

$$= \pi\int_0^1 (3(x-1)^3)^2$$
$$= \pi\int_0^1 9(x-1)^5\,dx$$

There were candidates who made careless mistakes in the last line, π was left out.

$$= \pi\left[\frac{4(x-1)^7}{7}\right]_0^1$$
$$= 0 - \left(-\frac{4}{7}\right)$$
$$= \frac{4}{7}\ \text{unit}^3$$

Question 11(a)

Most of the candidates who answered this question, could obtain the value of p = 0.4 from the statement `2 out of 5' in the question. They could get the value of q(q = 0.6) and used the formula ${}^{n}C_{r}\,p^{r}q^{n-r}$ with n = 8 and r = 2 of the binomial distribution to find the probability.

Some candidates could interpret the statement `probabality that more than 2' to mean $P(X>2)=1-P(X=0)-P(X=1)-P(X=2)$. There were a few who used $P(X>2)=P(X=3)+P(X=4)+\ldots+P(X=8)$.

a) $p = \frac{2}{5} = 0.4$

$q = 0.6$ P1

i) $= {}^{8}C_{2}(0.4)^{2}(0.6)^{6}$

$= \underline{0.2090}$ N1

ii) $1-(x=2)+(x=1)+(x=0)$

$= 1-(0.2090 + {}^{8}C_{1}(0.4)^{1}(0.6)^{7} + {}^{8}C_{0}(0.4)^{0}(0.6)^{8}$ K1

$= 1-(0.2090+0.089583+0.01680)$

$= \underline{0.6846}$ N1

However some candidates did not know that $X>2$ exclude X=2. Therefore they used $P(X>2)=1-P(X=0)-P(X=1)$

(ii) $1 - P(x=0) - P(x=1)$

$= 1-\left[{}^{8}C_{0}\left(\frac{2}{5}\right)^{0}\left(\frac{3}{5}\right)^{8}\right]-\left[{}^{8}C_{1}\left(\frac{2}{5}\right)^{1}\left(\frac{3}{5}\right)^{7}\right]$

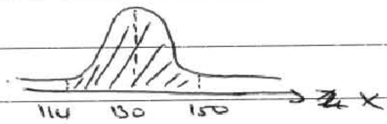
$= 1-[1\times1\times0.01679616]-[8\times(2/5)\times(2187/78125)]$

$= 1-0.01679616-0.08957952$ K0

$= 0.89362$ N0

Question 11(b)

Some candidates knew $\mu=130$ and $\sigma=16$ and used $Z=\dfrac{X-\mu}{\sigma}$, to find Z- score for X = 114 and X = 150. They knew how to use the Normal Distribution table or the scientific calculator to find the probability of Z-score. Hence they could find the probabality between 114 and 150 to be $1-P(Z.>1.25)-P(Z<-1)$. A few candidates could find the total number of workers by dividing 132 by the probabality obtained from (b) (i).



~~Probability~~ : $Z = \dfrac{150 - 130}{16}$ ✓M1

$= 1.25$ ← higher than 150

$P = 0.1057$

$Z = \dfrac{114 - 130}{16}$

$= -1$ ← lower than 114

$= 0.1587$

$\therefore$ Probability between 114 − 150 mmHg

$= 1 - (0.1057 + 0.1587)$ ✓M1

$= 0.7356$ # ✓N1

$\dfrac{132}{x} \times 1 = 0.1057$ ✓M1

$\therefore x = 1248.82$

≈ 1249 workers # ✓N1

OR

$\dfrac{132}{0.1056} = 1250$ workers # ✓M1 N1

Some candidates failed to find the correct probabality as shown in (i). Some managed to shade the correct region but could not find the correct probabality as shown in (ii).

(i)

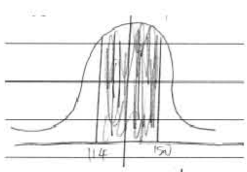
$1 + 0.1587 - 0.1057$

$= 1.053$

(ii)

$P(1 < x < 1.25) = \cancel{0.015}\ 0.1587 + 0.1056$

$= 0.2643$ # N0



A few candidates were careless in their calculation.

$= 1 - (0.1587 + 0.1056)$

$= 0.2643$ #

Candidates gave an approximate value when finding the probabality. The value of the probabality should be given to at least four decimal places as in the four-figure table.

$1 - 0{\cdot}159 - 0{\cdot}106$

$0{\cdot}735$ ✱

Question 12

Candidates who answered this question, were able to find the initial velocity of the particle by substituting t = 0 into v. Candidates could also differentiate the velocity, v, to obtain acceleration, a, and used a = 0 to find the minimum velocity. Candidates knew that when the particle moved to the left, $v < 0$ and hence they were able to solve the quadratic inequality to get the range of t.

Candidates knew how to integrate velocity to find displacement, s, and at least able to substitute t = 2 or t = 4 into s. Only candidates in the high achievement group managed to find the correct total distance travelled by the particle in the first 4 seconds using $s_2 + |s_4 - s_2|$ or $\int_0^2 v\,dt + \left|\int_2^4 v\,dt\right|$ or $2s_2 - s_4$.

12(a) $v = t^2 - 6t + 8$

$= 0^2 - 6(0) + 8$

$= 8\ m\,s^{-1}$ //

(b) $a = 0$

$\frac{dv}{dt} = 0 = 2t - 6$

$2t - 6 = 0$

$t = 3\ s$

$v = t^2 - 6t + 8$

$= 3^2 - 6(3) + 8$

$= -1\ m\,s^{-1}$ ✱

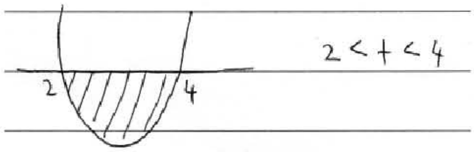
(c) $v < 0$

$t^2 - 6t + 8 < 0$

$(t-4)(t-2) < 0$

$t - 4 = 0$ @ $t - 2 = 0$

$t = 4$ $\quad t = 2$



$2 < t < 4$

(d) $s = \int v$

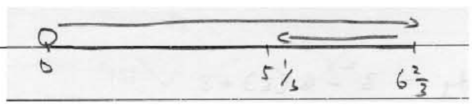
$= \int t^2 - 6t + 8$ /K1

$= \left[\frac{t^3}{3} - 3t^2 + 8t\right] + c$

when $s = 0,\ t = 0$

$\therefore c = 0$.

$\therefore s = \frac{t^3}{3} - 3t^2 + 8t$



Total distance $= 6\frac{2}{3}\ m + (6\frac{2}{3} - 5\frac{1}{3})\ m$

$= 8\ m$ /N1

Some candidates had the wrong concept on initial velocity, minimum velocity and the concept of particle moving to the left. They mixed up all the concepts.

(i)

initial velocity, $a = 0$ ~~$r\ 0$~~

$\frac{dt}{dv} = 0$

$= t^2 - 6t + 8$

$= 2t - 6$

$2t = 6$

$t = 3$

$v = 3^2 - 6(3) + 8$

$= -1$

Initial velocity means the velocity when $t = 0$

(ii)

minimun velocity, $s = 0$

$\cancel{\frac{ds}{dx} = 0}$

$\int t^2 - 6t + 8$

$= \int \frac{t^{2+1}}{3} - \frac{6t^{1+1}}{2} + 8$

$= \int \frac{t^3}{3} - 3t^2 + 8$

$= \frac{t^3}{3} - 3t^2 + 8t$

Minimum velocity means acceleration, $a = \frac{dv}{dt} = 0$

(iii)

c) $\int_0^t v\,dt < 0.$ When left, $s < 0$

$\int_0^t (t^2 - 6t + 8) < 0.$

Moving to the left means $v < 0$ NOT $s < 0$

(iv)

$s < 0$

$\frac{t^3}{3} - 3t^2 + 8t < 0$

$t^3 - 9t^2 + 24t < 0$

$t(t^2 - 9t + 24) < 0$

$\cancel{t(t} \quad t\,((t-6)(t-3)) < 0$

$3 < t < 6$

(v)

$\frac{t^3}{3} - \frac{6t^2}{2} + 8t < 0$

$2t^3 - 18t^2 + 48t < 0$

$t(2t^2 - 18t + 48) < 0$

$t(t - 4/5)(t - 4/5) < 0$

$\therefore t > 4/5$ s

(vi)

(c) when particle moves to the left, $a < 0$

$a = 2t - 6 < 0$ K0

$2t - 6 < 0$

$2t < 6$

$t < 3$ M0

Moving to the left means $v < 0$ NOT $a < 0$

A few candidates assumed initial velocity was zero.

(i)

Initial velocity = 0 m s^{-1} ko

(ii)

$V = t^2 - 6t + 8$

i) a) ~~$0 = (0)^2$~~ $v = 0$

A few candidates used a wrong method to get the value of t before finding the minimum velocity.

b) $\frac{dy}{dx} = \frac{t^3}{3} - 6t^2$ min velocity $= \frac{dy}{dx} = 0$

$0 = t - 6$ ko $v = (6)^2 - 6(6) + 8$ ko

~~$-6 = 0 - 6$~~ $6 = t$ $v = 36 - 36 + 8$

$v = 8$ ms^{-1}. NO

A few candidates assumed the minimum velocity was when t = 0.

(i)

$V = t^2 - 6t + 8$

minimum velocity, $t = 0$

$\frac{dv}{dt} = 2t - 6$

~~$V = 0^2 - 6(0) + 8$~~ $\frac{dv}{dt} = 6$ ms^{-1} ko ko NO

(ii)

when minimum velocity, $a = 0$, $t = 0$

$0 = 2t - 6$

$0 = 2(0) - 6$ ko ko

$0 = -6$ m s^{-1}

$\therefore V = -6$ m s^{-1} NO

Some candidates made a wrong conclusion to determine the range of t, giving the answer inclusive of t = 2 and t = 4 whereas the range did not include both values. The answer should be $2 < t < 4$.

$v = 0$

$0 = t^2 - 6t + 8$

$(t - 4)(t - 2) = 0$

$t = 4$ $t = 2$

$2s \leq t \leq 4s$ 1

A few candidates were not able to solve the quadratic inequalities correctly.

(i)

$v < 0$

$t^2 - 6t + 8 < 0$

$(t - 4)(t - 2) < 0$ K1

$t < 4$ or $t < 2$ NO

(ii)

c). $0 = t^2 - 6t + 8$

$(t - 4)(t - 2)$

$t = 4$ or 2 second

(iii)

$t^2 - 6t + 8 < 0$

$(t - 2)(t - 4) < 0$ K1

$t = 2$, $t = 4$

~~$t > 4$~~ $t < 4$

~~$t < 2$~~ $t > 2$ NO

Most candidates who answered this question were not able to find the total distance travelled by the particle in the first 4 seconds correctly although they managed to integrate v to get s. They also managed to substitute t = 2 or t = 4 into s to get the displacement at that particular time.

(i)

(d) $s = \frac{t^3}{3} - 3t^2 + 8t$ ✓ K1

$t = 2$

$s = \frac{2^3}{3} - 3(2)^2 + 8(2)$ ✓ K1

$= 6.67$ m

$t = 4$

$s = \frac{4^3}{3} - 3(4)^2 + 8(4)$

≈ 5.33 ✓

Total distance $= 6.67 + 5.33$ K0

$= 12$ m. N0

(ii)

a). $\cancel{\int t^2 - 6t + 8\, dt}$

$s = \frac{t^3}{3} - 3t^2 + 8$ ✓ K1

when $t = 4$

$s = \frac{4^3}{3} - 3(4)^2 - 8(4)$ ✓ K1

$\frac{64}{3} - 48 - 32$

$= -58\frac{2}{3}$ m

∴ total distance travelled $= 58\frac{2}{3}$ m K0

$= 58.67$ m N0

(iii)

$\int_0^4 t^2 - 6t + 4\, dt$

$= \left[\frac{t^3}{3} - 3t^2 + 8t\right]_0^4$ ✓ K1

$= \left(\frac{64}{3} - 48 + 32\right) - (0)$ K1

$= 5\frac{1}{3}$ m K0N0

(iv)

(d) $S = \int v\, dt$

$= \int (t^2 - 6t + 8)\, dt$

$= \frac{t^3}{3} - 3t^2 + 8t + c$ K1

when $s = 0$, $t = 0$, $\therefore c = 0$

$\therefore s = \frac{t^3}{3} - 3t^2 + 8t$

when $t = 4$,

$s = \frac{(4)^3}{3} - 3(4)^2 + 8(4)$ K1

$= 21.3 - 48 + 32$

$= 5.3$m

K0 N0

(v)

t	0	1	2	3	4
S	8	$\frac{40}{3}$	$\frac{44}{3}$	14	$\frac{40}{3}$

✓ K1

Total distance $= 8 + \frac{44}{3} + \left(\frac{44}{3} - \frac{40}{3}\right)$

$= 24$ metres.

(vi)

Total distance is ~~A~~ $6\frac{2}{3} + 5\frac{1}{3} = 12$m

A few candidates did not integrate all the terms from the function, v, for example, leaving out integrating the constant ‘8’ in this case.

d) $S = \frac{t^3}{3} - 3t^2 + 8$ ✓ K1

There were still candidates who made errors in their calculation.

(i)

$= \frac{t^3}{3} - \frac{6t^2}{2} + 8t + c$

$= \frac{(4)^3}{3} - \frac{6(4)}{2} + 8(4) + c$

$= 41.3$

(ii)

$s = \frac{4^3}{3} - 3(4)^2 + 8(4)$

$= 21\frac{2}{3} - 48 + 32$

$= 5\frac{2}{3}$

total distance travelled in first four

$\Rightarrow 6\frac{2}{3} + [6\frac{2}{3} - 5\frac{2}{3}]$

$= 7\frac{2}{3}$ m. N0

(iii)

$\therefore$ total distance $= 6\frac{2}{3}$ m $+ (6\frac{2}{3}$ m $- 5\frac{1}{3})$ K1

$= 5\frac{1}{3}$ m N0 $13\frac{1}{3} - 5\frac{1}{3}$

$= 5.33$ m